
ECONOMY LOADING

of Power Plants and Electric Systems

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To

EDWARD C. M. STAHL

in appreciation of his inspiration,
counsel, and encouragement

PREFACE

During the past twenty years progress in the power-generation field and the interconnecting of large electric systems have created the problem of economy loading. The allocation of load to power plants and to the equipment within them greatly influences the production costs, and the method of loading should be such that electric energy is produced at the lowest possible cost consistent with the obligation of maintaining continuity of service. The importance of this subject is evidenced by the volume of literature written on it, some of which is listed in the Bibliography.

Economy loading is obtained by the application of what is now generally known as the "incremental rate theory." Although acceptance of this theory is general, many misconceptions still exist in respect to its practicability and the methods of applying it. In the preparation of the material for this book we have therefore employed the following pattern.

1. The derivation of the mathematical conditions for obtaining maximum overall efficiency by the loading of equipment in parallel operation.
2. The application of incremental rates for the academic solution of load-division problems.
3. Limitations in the application of the incremental rate theory.
4. The practical solution of load-division problems for the purpose of eliminating laborious computations without materially affecting the precision of the results.

The material in this book has been developed to meet the requirements of several groups whose interests lie in the field of power generation and to whom economy loading is a problem directly or indirectly associated with their specific functions. The subject matter is of primary interest to load dispatchers, system operators, and operators of generating station equipment, because economy loading is an ever-present problem to them. Planning engineers will find the material useful in evaluating the relative merits of alternative proposals for the addition of generating equipment to a system. As the economy aspects of power-plant design and operation are being increasingly stressed in engineering schools, it is hoped that this volume will be useful as a supplementary textbook for courses dealing with heat-power and power-plant engineering, and as a reference book in the general field of engineering economics.

We have used numerous illustrations to demonstrate the practical application of the principles discussed. The illustrations are simple, and we believe that they make it possible for the reader to understand and to apply incremental loading under the particular conditions which prevail in his system even though he may not wish to make use of the mathematical material.

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CHAPTER I

THEORETICAL BACKGROUND

Introduction

Economy of production is one of the most important phases of operating practice with respect to the generation, transmission, and distribution of electric energy. The ability to attain and maintain production economy depends in a large measure upon the methods of allocating load, not only to the equipment in the power plant, but also to the generating stations of a single system and to entire systems interconnected through high-tension transmission lines. In the power plant, the operator is confronted with the problem of selecting the equipment to be put in service and then of operating the equipment to obtain the greatest possible efficiency of production. The system load dispatcher, although primarily concerned with providing sufficient capacity to maintain continuity of service, is also concerned with the problem of allocating the load among the generating stations under his jurisdiction so as to obtain the highest system efficiency of production. The interconnection of electric systems has resulted in the problem of determining when to interchange energy, how much energy to interchange, the cost of supplying the energy to the interconnection, and the value of the energy received from the interconnection. How these problems may be solved in a practical manner consistent with theoretically correct principles constitutes the subject matter of this book.

That load division between two or more machines does affect the overall economy of operation is easily demonstrated. Consider two machines operating in parallel to supply a common load. The term "machine" applies to any apparatus or combination of devices capable of changing energy from one form into another. Depending upon the nature of the machine, there will be a definite relation between its input and output, which can be graphically established as an input-output curve. For the purpose of illustration the input-output curves for two machines are shown in Fig. 1. For any given combined output, the problem is to determine what the output of each machine should be in order that the combined input shall be at a minimum value. This can be done by a cut-and-try process. For example, suppose that the total

output is 12; then either machine could operate at any output ranging in value from 2 to 10, while the other machine supplied the remainder of the load. By tabulating the possible combinations of machine outputs

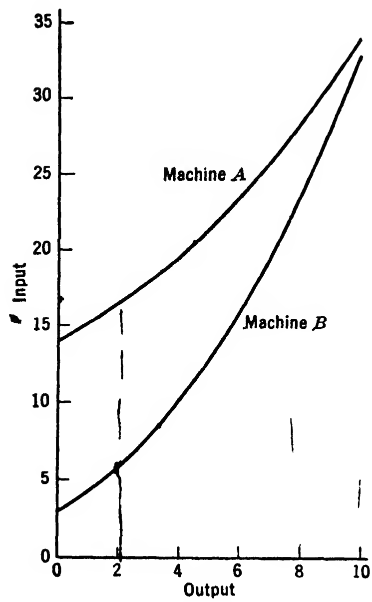


FIG. 1. Machine input-output curves.

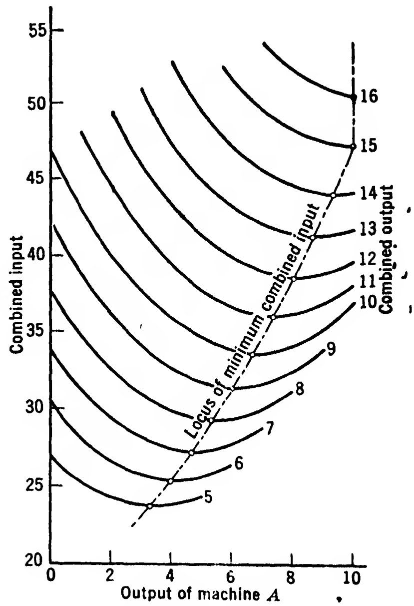


FIG. 2. Combined input to machines A and B of Fig. 1 plotted as a function of machine A output.

and the corresponding total inputs to both machines, the combination for the most efficient operation can then be selected, as shown by the following:

Output Machine A	Output Machine B	Input Machine A	Input Machine B	Input Total
2	10	16.4	33.0	49.4
3	9	17.9	28.2	46.1
4	8	19.6	23.8	43.4
5	7	21.5	19.8	41.3
6	6	23.6	16.2	39.8
7	5	25.9	13.0	38.9
8	4	28.4	10.2	38.6 (Minimum)
9	3	31.1	7.8	38.9
10	2	34.0	5.8	39.8

From the foregoing it is seen that, for the combined output of 12, the outputs of machine A and machine B should be respectively 8 and 4, and

that for this combination the input will be less than for any other combination of outputs.

By repeating this process for other values of combined output, a loading schedule can be established to cover the entire range. Figure 2 illustrates the effect of load division on the efficiency of operation. The horizontal scale indicates the output of machine A. The vertical scale indicates the combined input. The circles on the curves indicate the minimum values of combined input, for which the locus is the dotted curve. From the values represented by the circles, the load division between the two machines can be graphically represented as shown in Fig. 3.

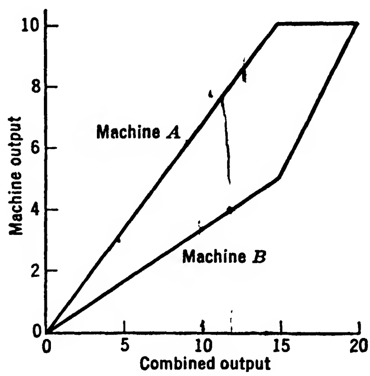


FIG. 3. Loading of machines A and B of Fig. 1 for maximum combined efficiency.

This process, although practicable for application to two machines, becomes laborious for application to more than two. The problem of load division can be solved much more readily by the application of incremental rates. The procedure involved is based on sound theory and can be applied with facility regardless of the number of machines involved.

Concept of an Incremental Rate

The incremental rate of a machine at any given output is numerically equal to the slope of the input-output curve at the point corresponding to that output. Mathematically, the incremental rate is the first derivative of the input-output curve with respect to the output. It measures the *rate of change* of the input with respect to the output, and does not indicate the absolute value of the input. For example, referring to the input-output curve for machine A in Fig. 1, at an output of 4 the input is 19.6. If the output is increased to 5, the input is increased to 21.5. For an *increase* of 1 in the output there is a corresponding *increase* of 1.9 in the input. The *increase* in output is called the *incremental* output, and the *increase* in input is called the *incremental input.* The incremental rate may be defined as the ratio[1] of the incremental input to the incremental output or

$$\text{Incremental rate} = \frac{\text{Incremental input}}{\text{Incremental output}} = \frac{1.9}{1.0} = 1.9$$

[1] More exactly, the incremental rate is the *limit* approached by the ratio of the incremental input to the incremental output as the latter approaches zero as a limit.

The incremental rate of 1.9 is an *average* value for the increment of output between 4 and 5.

The incremental efficiency is the reciprocal of the incremental rate or

$$\text{Incremental efficiency} = \frac{1}{\text{Incremental rate}} = \frac{\text{Incremental output}}{\text{Incremental input}}$$

$$= \frac{1}{1.9} = 0.536 = 53.6 \text{ per cent}$$

At an output of 4, the absolute input from the curve is 19.6. The *absolute efficiency* is 4/19.6 = 0.204 = 20.4 per cent.

It is important to note the distinction between the *absolute efficiency* and *incremental efficiency.* The absolute efficiency merely gives the ratio between the output and the corresponding input. The incremental efficiency gives the efficiency at which an increment of output will be obtained and will determine which machine should supply an *increase* in load. This is illustrated by the following example.

	Boiler *A*	Boiler *B*
Boiler output, million Btu per hour	10	10
Incremental rate	1.42	1.33
Boiler efficiency, per cent	80	70

If it were necessary to increase the total output by 1 million Btu per hour, then the additional or incremental output should be supplied by boiler *B*, notwithstanding the fact that the absolute efficiency of boiler *A* is better than that of boiler *B*. The criterion is not the relative boiler efficiencies but the relative efficiencies of generating the *additional* or *incremental* output. To supply the incremental output of 1 million Btu per hour from boiler *A* would require an incremental input of 1.42 million Btu per hour, and only 1.33 million Btu per hour for boiler *B*. Hence the incremental output should be supplied by boiler *B* even though it is the less efficient boiler. The incremental efficiencies with which the incremental output could be supplied by each boiler are

$$\text{Boiler } A \qquad \frac{1}{1.42} \times 100 = 70.42 \text{ per cent}$$

$$\text{Boiler } B \qquad \frac{1}{1.33} \times 100 = 75.19 \text{ per cent}$$

Calculation of Incremental Rates

The incremental rate may be derived by one of the following methods:

1. When the input-output curve can be expressed by an algebraic equation, the incremental rate can be determined by differentiation, since it is the first derivative of the equation for the input-output curve.

Thus the equations for the input-output curves of Fig. 1 may be algebraically expressed as

$$I_a = 14 + L_a + 0.1L_a^2 \qquad [1]$$

$$I_b = 3 + L_b + 0.2L_b^2 \qquad [2]$$

where I = input, L = output, and the subscripts refer to the machines.

The incremental rates for the machines can then be expressed as

$$\frac{dI_a}{dL_a} = R_a = 1 + 0.2L_a \qquad [3]$$

$$\frac{dI_b}{dL_b} = R_b = 1 + 0.4L_b \qquad [4]$$

where R_a and R_b are the incremental rates for machine A and machine B, respectively.

2. The incremental rate may be graphically determined by drawing a tangent to the input-output curve at the point corresponding to the output. The slope of the tangent is equal to the incremental rate at the given output. The method is illustrated in Fig. 4. At point A on the input-output curve, corresponding to the output L, the tangent ab is drawn. From any two points on the tangent, such as c and d, lines are drawn parallel to the horizontal and vertical axes, respectively. These will intersect at some point e. The slope of the input-output curve (or the incremental rate) at point A on the curve is obtained by dividing ed by ce.

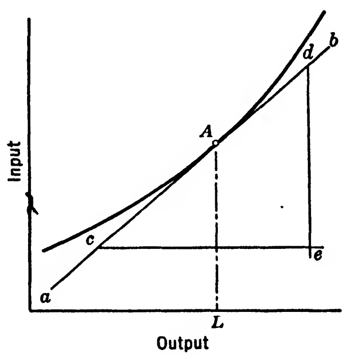


FIG. 4. Graphical determination of incremental rates.

3. From the input-output curve, a series of output values are chosen and the corresponding input values are read. The difference between successive values of the output are usually made constant and are made small enough so that the characteristic shape of the incremental rate curve can be determined with reasonable accuracy. The incremental rate is merely the ratio of the input difference to the output difference,

or the incremental input divided by the incremental output, and is assumed to be a function of the midpoint.[1] This method is illustrated in Fig. 5 and Table I. It is to be noted that the method will not permit the determination of the end points of the incremental rate curve except by extrapolation.

Of the three methods discussed, the third is the most practical. The first is not recommended because algebraic equations would have but a

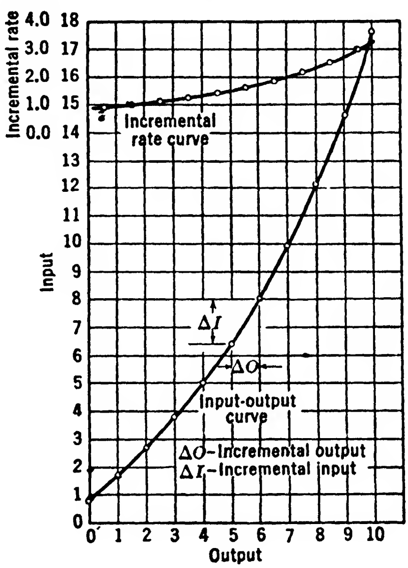


FIG. 5. Incremental rates plotted against mid-point values of incremental outputs.

very limited application and a considerable amount of time and effort would be required to fit them to the types of input-output curves generally associated with power-plant equipment. Furthermore, a few sample computations will disclose that the results obtained with the third method will be closely checked by those obtained with the first.

There are several reasons for not recommending the second method. First, it requires plotting of the input-output curve; and, as will be subsequently shown, there are many instances when this is unnecessary. Second, where the input-output curve is plotted, it will generally be found that the tangent cannot be established with any reasonable degree of accuracy. Hence the resulting values will be inconsistent and unsuit-

[1] This method assumes that a secant subtended by the arc of the input-output curve is parallel to the tangent at the midpoint of the arc. No significant err made by this assumption if sufficiently small output increments are used.

TABLE I

Output	Incremental Output	Input	Incremental Input	Incremental Rate	Output against Which Incremental Rate is Plotted
0		0.8			
	1.0		0.9	0.9	0.5
1		1.7			
	1.0		1.0	1.0	1.5
2		2.7			
	1.0		1.1	1.1	2.5
3		3.8			
	1.0		1.2	1.2	3.5
4		5.0			
	1.0		1.4	1.4	4.5
5		6.4			
	1.0		1.6	1.6	5.5
6		8.0			
	1.0		1.9	1.9	6.5
7		9.9			
	1.0		2.2	2.2	7.5
8		12.1			
	1.0		2.5	2.5	8.5
9		14.6			
	1.0		3.0	3.0	9.5
10		17.6			

able for determining the characteristic shape of the incremental rate curve.

Conditions for Maximum Efficiency

Considering first the general case, it may be stated that, when two or more machines are operating in parallel to supply a common load, the maximum overall efficiency, i.e., the minimum combined input for the given combined output, is obtained when the machines are operating at outputs which correspond to the same incremental rate value. This is based upon three conditions, namely:

1. That the input-output curves are continuous.
2. That the first derivatives of the input-output curves (the incremental rate curves) are continuous.

That the value of incremental rate *always* increases as the output increases.

The proof of this theorem, applied to two machines, follows:

Let O_t = total output of the two machines.
O_1 = output of machine 1.
O_2 = output of machine 2.
I_t = total input to the two machines.
I_1 = input to machine 1.
I_2 = input to machine 2.

It is assumed that $I_1 = f_1(O_1)$ and $I_2 = f_2(O_2)$ represent two continuous input-output curves and have continuous derivatives $\frac{dI_1}{dO_1}$ and $\frac{dI_2}{dO_2}$, respectively, which always increase as O_1 and O_2 increase. Therefore, the second derivatives $\frac{d^2I_1}{dO_1^2}$ and $\frac{d^2I_2}{dO_2^2}$, and hence their sum, are always positive.

$$I_t = I_1 + I_2 \tag{5}$$

and

$$O_t = O_1 + O_2 \tag{6}$$

The problem is to determine, for any given value of O_t, for example, O_a, the value of O_1 and O_2 which will make I_t a minimum. Then

$$O_1 + O_2 = O_a \qquad \text{a constant} \tag{7}$$

$$O_2 = O_a - O_1 \tag{8}$$

$$I_t = I_1 + I_2 \tag{9}$$

I_t will be a minimum when its first derivative with respect to O_1 vanishes, since (as will be shown later) its second derivative is positive. Thus

$$\frac{dI_t}{dO_1} = 0 \tag{10}$$

but

$$\frac{dI_t}{dO_1} = \frac{dI_1}{dO_1} + \frac{dI_2}{dO_1} \tag{11}$$

$$= \frac{dI_1}{dO_1} + \frac{dI_2}{dO_2} \times \frac{dO_2}{dO_1} \tag{12}$$

Furthermore

$$\frac{dO_2}{dO_1} = \frac{d(O_a - O_1)}{dO_1} = -1 \tag{13}$$

Hence

$$\frac{dI_t}{dO_1} = \frac{dI_1}{dO_1} - \frac{dI_2}{dO_2} \qquad [14]$$

Equating 10 and 14,

$$\frac{dI_1}{dO_1} - \frac{dI_2}{dO_2} = 0 \qquad [15]$$

or

$$\frac{dI_1}{dO_1} = \frac{dI_2}{dO_2} \qquad [16]$$

This means that the minimum input for a given combined output is obtained when the incremental rates of the two units are equal.[1] Furthermore, there is only one pair of loads for any given combined output at which the incremental rates will be equal.

The incremental rate curves corresponding to the input-output curves of Fig. 1 are shown in Fig. 6. The incremental rate curve for the two

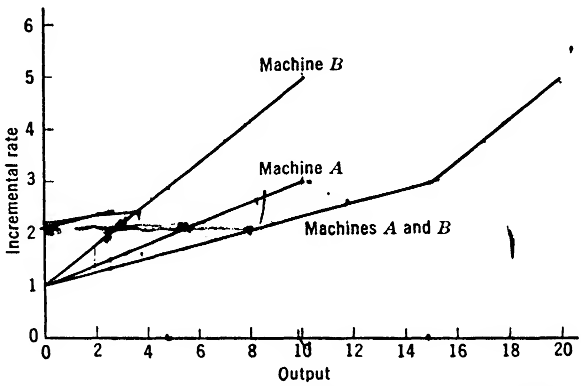


FIG. 6. Incremental rate curves for machines of Fig. 1.

machines combined, obtained by adding the outputs of the individual machines at any incremental rate value and plotting the total output at the same incremental rate value, is also shown. Thus, at an incremental rate of 2.0, the outputs of machines A and B are respectively 5.0 and 2.5, so that the combined output at that incremental rate is 7.5. From these curves the load division derived from Fig. 2 can be readily checked.

[1] By differentiating 14,

$$\frac{d^2I_t}{dO_1^2} = \frac{d}{dO_1}\left[\frac{dI_1}{dO_1} - \frac{dI_2}{dO_2}\right] = \frac{d^2I_1}{dO_1^2} - \frac{d}{dO_2}\left(\frac{dI_2}{dO_2}\right)\frac{dO_2}{dO_1} = \frac{d^2I_1}{dO_1^2} - \frac{d^2I_2}{dO_2^2}(-1) = \frac{d^2I_1}{dO_1^2} + \frac{d^2I_2}{dO_2^2}$$

Since both parts of the sum are positive, the sum is positive and corresponds to a minimum.

For example, at a combined output of 12, the incremental rate is 2.6, and for this value the outputs of machines A and B should be 8 and 4, respectively.

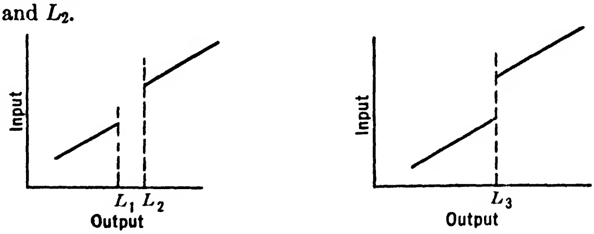
It is to be noted that the use of incremental rate curves to determine the proper load division between the machines does not require any tabulation of input values. This accounts for the simplicity of this method compared with the cut-and-try process illustrated by the curve net of Fig. 2.

Characteristics of Input-Output Curves

It is not generally recognized that there are limitations to the application of incremental rates to load division. Frequently it is erroneously concluded that the division of load among a group of machines so that each is operating at an output corresponding to the same incremental rate value will result in the best overall performance under all conditions. Actually, there are circumstances under which such procedure will result in the worst rather than the best overall efficiency. To understand when loading at equal incremental rates should not be applied, it is necessary to analyze the characteristics of the input-output and corresponding incremental rate curves.

Input-output curves may be classified as being either continuous or discontinuous. The terms "continuous" and "discontinuous" are used in their generally accepted mathematical sense. If the input-output curve is continuous, then for any output the corresponding input will have a single finite value. An input-output curve is discontinuous if there is any output at which there are two corresponding values of input; if there is a range of output for which there are no corresponding values of input; or if at any output there is a sudden change in the input.[1]

[1] If the input-output curve is discontinuous, it may mean that there is a value of the output for which no input value exists, as shown in Fig. (a) in the range of output between L_1 and L_2.



More likely it may mean that there is an output value at which there is a step-like increase or decrease in the input as shown in Fig. (b) at an output of L_3. Mathematically this condition can be described by saying that the limit approached by the input, as the output value of L_3 is approached, depends upon the direction from which the value of L_3 is approached.

Continuous Input-Output Curves

The types of continuous input-output curves which are generally encountered in power-plant calculations are illustrated in Fig. 7. Although these are continuous curves, it will be noted that their corresponding incremental rate curves vary in their mathematical characteristics.

Referring to Fig. 7, the Type I curves satisfy the three conditions upon which the general rule is based. The input-output curve is continuous, and its incremental rate curve is continuous with values that always increase as the output is increased.

The Type II input-output curve is a smooth continuous curve. The distinguishing feature is the point of inflection shown at point *a* on the curve. For outputs up to the value corresponding to point *a*, the incremental rate values *decrease* as the output increases, and hence the third condition under the general rule is violated. Because of this characteristic, it will subsequently be shown that the incremental rates are not the only criteria for determining the proper load division between machines having this type of input-output curve.

Input-output curves consisting of three intersecting straight lines are shown as Types III and IV. The incremental rate values are constant for each section, and at each point of discontinuity there are two incremental rate values. Input-output curves consisting of three intersecting smooth curves are shown as Types V and VI. The incremental rate values are not constant; furthermore, there are two incremental rate values at each point of discontinuity. Because the incremental rate curves are not continuous, these types do not come within the general rule. It is to be noted that the incremental rate values increase at the points of discontinuity for the Type III and Type V curves, and decrease for the Type IV and Type VI. The nature of the change in value of the incremental rate at a point of discontinuity is very important with respect to the determination of the proper load division.

With the foregoing as a background, consideration will now be given to the principles underlying the procedure to determine the proper load division for machines whose continuous input-output curves do not meet all three of the conditions under the general rule.

Considering first machines with Type III input-output curve characteristics, it is to be noted that, although the corresponding incremental rate curve is discontinuous, the incremental rate values increase at the points of discontinuity. Hence, there are no decreasing values of incremental rate as the output is increased. If at the point of discontinuity the incremental rate is considered to have *all* the values between the lower and upper limits, it is possible to operate the machines at outputs

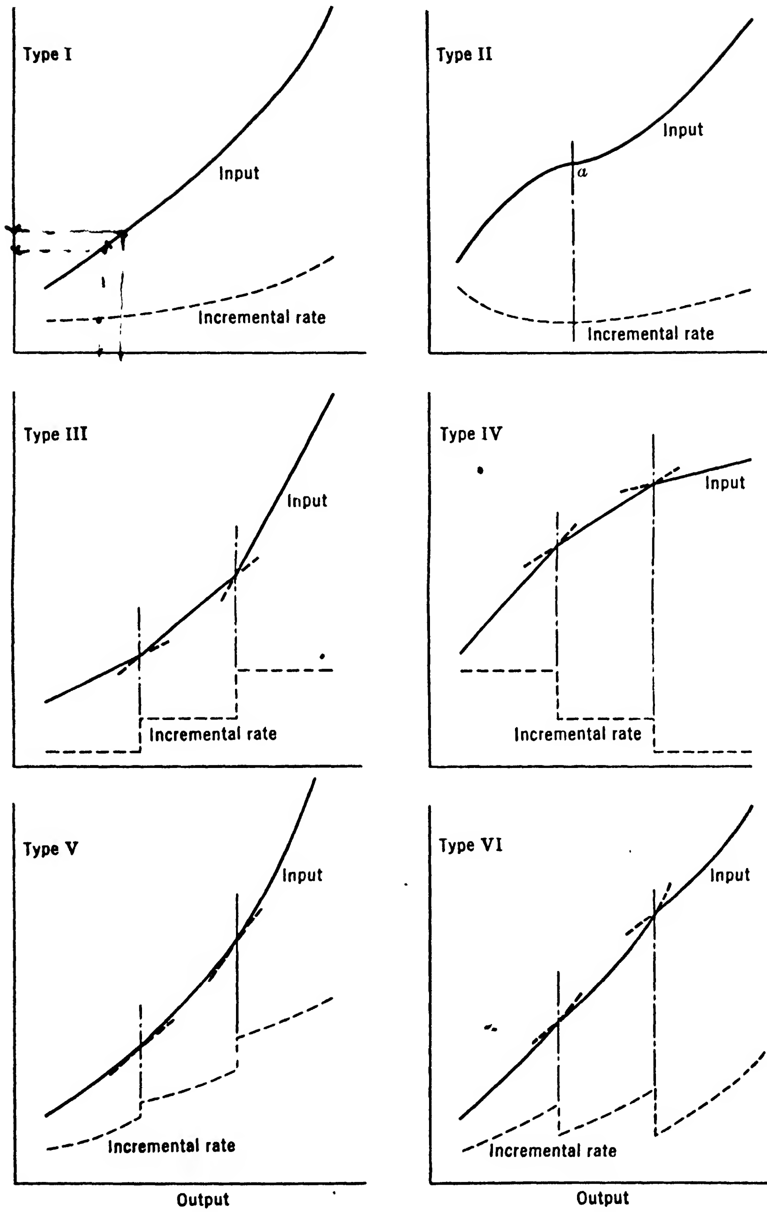


FIG. 7. Typical continuous input-output curves and corresponding incremental rate curves.

corresponding to the same incremental rate value, and the curve comes within the general rule. The mathematical proof for the general case indicates that for any combined output the combined input to two machines will be a minimum for only *one* combination of loads on the individual machines. Under the interpretation given to the incremental rate at the point of discontinuity, this condition prevails except in the rare case where the incremental rate curves have ranges in which the incremental rate values are equal as well as constant. For such a condition there will be an infinite number of combinations for any total output within the given range, for which the combined input will be constant and at a minimum value. These principles are illustrated by the curves of Fig. 8.

Referring to Fig. 8, the input-output curves A and B, and the corresponding incremental rate curves AA and BB, are similar to Type III of Fig. 7. Up to a combined output of 7, machine B is maintained at an output of 1 while machine A supplies the remainder of the load. This follows from the fact that, in this range of output, machine A has a lower incremental rate value than machine B. For combined outputs from 7 to 9, machine A is operated at an output of 6 while machine B supplies the rest. In this range, both machines would be operating at outputs corresponding to the same incremental rate value of 0.6. It is to be noted that, for any total output up to and including the value of 9, there is only one combination of loads for which the combined input is a minimum. This is indicated by the locus of minima for the curve net showing the combined input to machines A and B as a function of the output of machine A. For any total output between the values of 9 and 13, the load division between the machines will not affect the combined input, provided that the output of machine A is not less than 6 or more than 8, and that of machine B is not less than 3 or more than 5. The reason for this lies in the fact that the incremental rate of machine A for outputs between 6 and 8 is equal to the incremental rate of machine B for outputs between 3 and 5. Within these ranges, the combined input will not be affected by the machine which supplies the incremental output. Graphically this is illustrated by the plane area included in the parallelogram $KLMN$.

From the discussion so far some general conclusions may be drawn. These are:

1. If the incremental rate values of a continuous input-output curve do not decrease as the output increases, then the combined input to two or more machines whose individual curves have these characteristics will be a minimum when the individual machine outputs correspond to the same incremental rate value, and

2. For any combined output there can be only one combination of machine outputs for which the incremental rates of the individual machines are equal, with the exception that

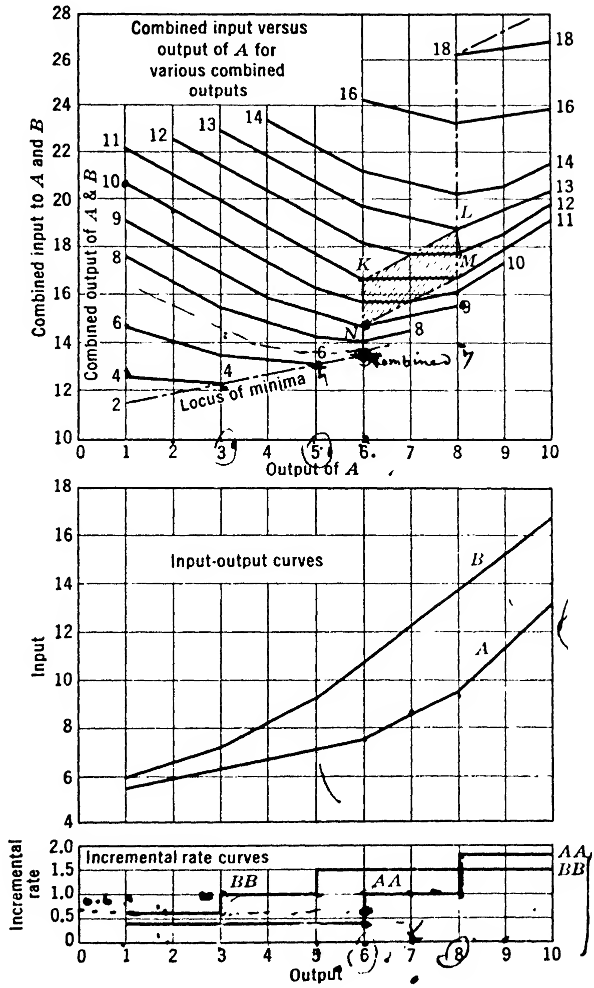


FIG. 8. Effect of load division on combined efficiency for machines having Type III (Fig. 7) input-output curves.

3. If, for two or more machines, there are ranges of output for which the incremental rates of the individual machines are equal and constant, then there may be an indefinite number of combinations of machine outputs for which the combined input will remain constant and be at a minimum value.

4. The above conclusions apply to input-output curves having the characteristics illustrated in Fig. 7 as Type I, Type III, or Type V.

By analysis of the curve nets of Figs. 2 and 8 other conclusions may be drawn about the characteristics of the combined input curves for machines having the input-output characteristics under discussion.

1. There is neither a point of mathematical maximum nor a point of inflection on any of the combined input curves shown as a function of the output of one machine.

2. The point of *actual minimum* combined input for any combined output *coincides* with the point of *mathematical minimum,* except when one of the machines is required to operate at either its minimum or maximum output to obtain the actual minimum combined input, under which conditions there is no point of mathematical minimum on the combined input curve and the machines cannot be operated at outputs corresponding to the same incremental rate value.

If one machine of a group has incremental rate values which decrease as the output increases, then, for any combined output for the group, there may be two or more combinations of machine outputs for which the respective incremental rates will be equal. The problem then becomes one of selecting the combination of machine outputs which will result in the *actual* minimum combined input.

This is illustrated by the curves of Fig. 9. The input-output and corresponding incremental rate curves of machines A and B are respectively similar in characteristics to those of Types I and II of Fig. 7. From inspection of the combined input curve net it is seen that some of the combined input curves have mathematical minima, some have both mathematical minima and maxima, and some have neither. At the combined output of 80, there are two combinations of machine outputs which correspond to equal incremental rate values. These are:

Combination	Incremental Rate	Output Machine A	Output Machine B	Output Total
1	1.7	71	9	80
2	1.2	32	48	80

The first combination of machine outputs corresponds to the mathematical maximum, and the second to the mathematical minimum, of the combined input curve. The mathematical minimum is also the actual minimum combined input.

It is further to be noted that for combined outputs greater than 69 the point of mathematical minima on the combined input curve indicates the load division for the best overall efficiency; i.e., the points of actual and mathematical minimum are coincident. For combined outputs

below that value, the actual minima are obtained when machine B is operated at minimum output *even though by so doing the machines are not operated at equal incremental rates.*

As indicated by the curve net of Fig. 9, it cannot be concluded as a general proposition that the points of mathematical and actual minima

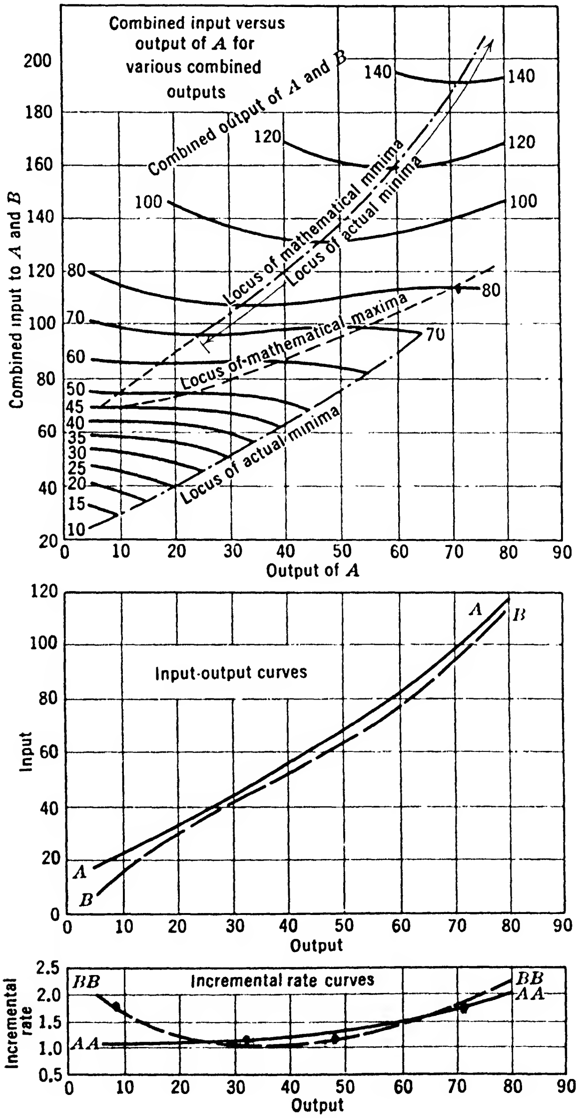


FIG. 9. Effect of load division on combined efficiency for machines having Types I and II (Fig. 7) input-output curves.

will be coincident. If it is definitely known, however, that these will be coincident, then it will be helpful to know how the mathematical minimum may be determined from the respective incremental rate curves.

It can be mathematically demonstrated that a point on the combined input curve at which the incremental rates of the several machines are equal is either a mathematical maximum, minimum, or point of inflection if the *sum of the slopes of the incremental rate curves* at the respective machine outputs is respectively negative, positive, or zero.[1] Thus at the combined outputs of 80, for the first combination of machine outputs, the slope of the incremental rate curve for machine A at an output of 71 is positive, and that for machine B at an output of 9 is negative. By inspection it is evident that the sum of the slopes is negative, and hence this combination of loading corresponds to a mathematical maximum on the combined input curve. As the mathematical maximum can never coincide with the actual minimum combined input, this combination of machine outputs can at once be rejected.

For the second combination, the slopes of the incremental rate curves AA and BB are both positive at the points corresponding to the respective machine outputs of 32 and 48. Hence the sum of the slopes is positive, and this combination of machine outputs therefore corresponds to a point of mathematical minimum on the combined input curve. In this instance the mathematical minimum is also the actual minimum.

Analysis of the curve net of Fig. 9 shows that the actual minimum combined input may occur for the combination of loading corresponding to the end point of the combined input curve, involving the operation of machine B at its minimum output. It is therefore important to understand under what conditions the end point will represent the actual minimum combined input.

First, when the combined input curve does not have a mathematical minimum, an end point will always correspond to the actual minimum combined input.

[1] If the sum of the slopes is zero, it is still possible to have a maximum or a minimum rather than a point of inflection. The rigorous test is as follows:

Determine the successive higher derivatives of the equation for the combined input curve until one is found which is not zero at the output in question. If this is an odd-numbered derivative, such as the third, fifth, or seventh, there is a point of inflection. If it is an even-numbered derivative, such as the fourth, sixth, or eighth, the point is a maximum if the derivative is negative and a minimum if the derivative is positive.

Since most of the functions dealt with in practice cannot be readily expressed as algebraic equations, the higher derivatives will not often be obtainable. Usually it will be discernible from the incremental rate curves whether a point is a maximum, minimum, or point of inflection.

Second, when a mathematical maximum lies between an end point and a mathematical minimum, that end point may be the actual minimum or be very close to it. Hence the end point loading should be checked against the loading corresponding to the mathematical minimum to determine which loading will give the actual minimum combined input.[1]

The relationship between the mathematical and actual minima on the combined input curve can be further illustrated by considering the problem of load division between two machines having input-output curves similar to those shown for Type VI of Fig. 7, for which the corresponding incremental rate curves have discontinuities at which the incremental rate values decrease. This problem is presented by the curves of Fig. 10. Referring to this figure, the combined input curve net indicates that there may be two or more mathematical minima on a curve. Further analysis indicates that, for each mathematical minimum, the outputs of the individual machines correspond to equal incremental rate values. Since the slopes of the respective incremental rate curves are positive, except at the points of discontinuity, it follows that the sum of the slopes will be positive and hence the corresponding points on the combined input curve must be mathematical minima. To illustrate, consider the possible incremental loading combinations for a combined output of 11.5, as tabulated below:

Combination	Incremental Rate	Machine Outputs A	Machine Outputs B	Machine Outputs Total	Combined Input
1	1.75	7.15	4.35	11.5	18.750
2	1.80	7.00	4.50	11.5	18.770
3	1.86	7.50	4.00	11.5	18.795
4	1.93	7.77	3.73	11.5	18.792
5	1.95	6.55	4.95	11.5	18.702

The above tabulation clearly demonstrates that, for the given combined output, it is possible to operate the two machines at equal incre-

[1] It may be stated in passing that, when the machine input-output curves and the corresponding incremental rate curves are continuous, the number of points on the combined input curve corresponding to mathematical maxima is within one of being equal to the number corresponding to mathematical minima, and maxima and minima alternate with each other (in other words, there cannot be either two successive maxima or minima). Thus if there are two mathematical minima it is possible for the curve to have one, two, or three maxima; if there is only one maximum, it must be between the two minima. For this case, the end point cannot correspond to the actual minimum combined input and may therefore be disregarded. Furthermore, if there is no maximum between a minimum and an end point, that end point cannot correspond to an actual minimum combined input and hence need not be checked.

mental rate values for five different combinations. Of these, combinations 2 and 3 may be immediately disregarded as corresponding to mathematical maxima on the combined input curve. This follows from the

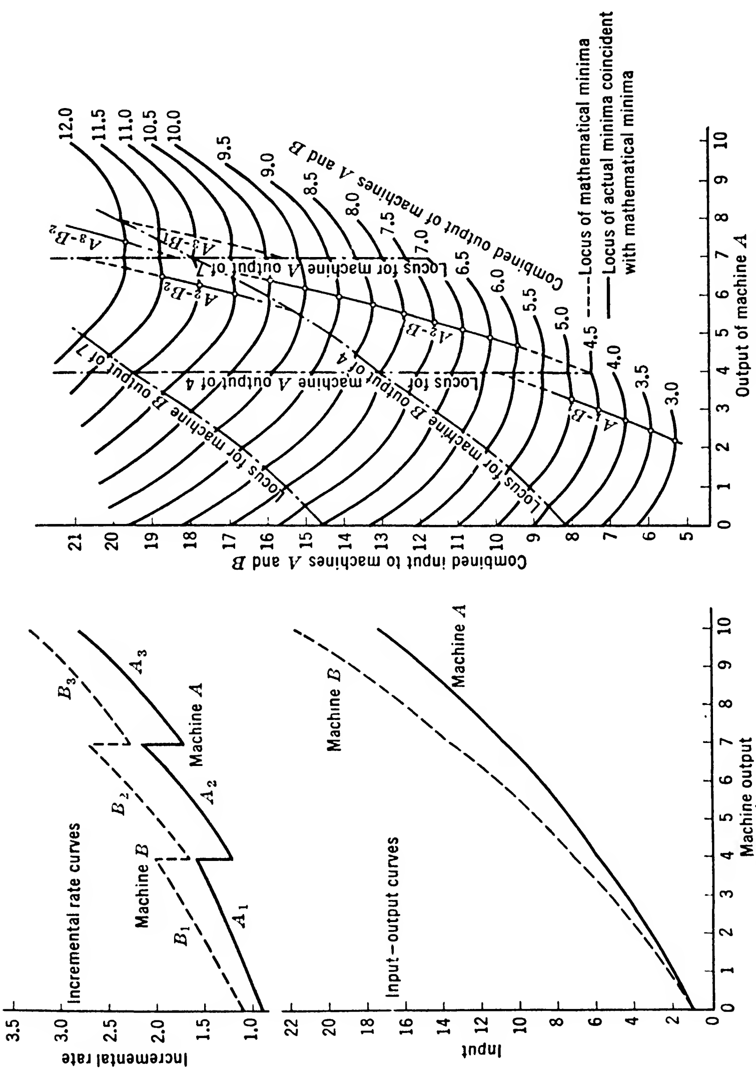


FIG. 10. Effect of load division on combined efficiency for machines having Type VI (Fig. 7) input-output curves.

fact that one of the machines for each combination would be required to operate at an output corresponding to a point of discontinuity on its respective incremental rate curve.[1]

[1] If the incremental rate curves are regarded as having at the points of discontinuity all the values between the upper and lower limits, they may be treated as

The remaining three combinations correspond to mathematical minima on the combined input curve. To determine which one of these corresponds to the actual minimum requires that the combined input for each combination be determined. The above tabulation shows that combination 5 corresponds to the actual minimum of the combined input curve of Fig. 10.

In order to cover the complete range of loading, the method illustrated in Fig. 11 can be used. The combined incremental rate curve is established from the incremental rate curves of the individual machines. Since these curves consist of three segments each, the combined curve will also consist of segments. For example, segment A_1B_1 of the combined curve is derived from segments A_1 and B_1 of machines A and B, respectively. In some ranges of combined output it is possible to have two or more combinations. Thus for combined outputs between 4.5 and 6.25, it is possible to have combinations represented by the segments A_1B_1 and A_2B_1; in the range of output between 9.75 and 10.65 it is possible to have three combinations, A_2B_1, A_2B_2, and A_3B_1. The points of transition from one combination to another for best overall performance are obtained by plotting the overall efficiency for each combination against the total or combined output. As shown in Fig. 11, the efficiency curves for the successive combinations will intersect, and the total load at the point of intersection is the load at which it becomes economical to pass from one combination to the other.

Referring to Fig. 11, the combined incremental rate curve with the points of transition for maximum overall economy is shown by the full line. The portions shown by the dotted lines correspond to loading at equal incremental rates and to mathematical minima on the combined input curves, but they do not correspond to actual minima.

Determination of the points of transition by the method described above requires a considerable amount of calculation when more than two machines are involved. In the interest of simplification it is frequently expedient to resort to the use of average incremental rate values when they will give results within an acceptable degree of accuracy. To illustrate how this can be applied to the determination of the transition

though they were continuous. The operation of one machine at an output corresponding to a point of discontinuity on its incremental rate curve, and the other machine at an output corresponding to an incremental rate which lies between the upper and lower limits at the point of discontinuity for the first machine, may be considered to correspond to operation of both machines at equal incremental rates. Since the slope of the incremental rate curves in question is $-\infty$ at a point of discontinuity, the sum of the slopes, or second derivatives, for this combination of outputs must be negative, and hence the point on the combined input curve must correspond to a mathematical maximum.

points for the two machines under discussion, portions of their respective incremental rate curves have been reproduced in Fig. 12. Referring to

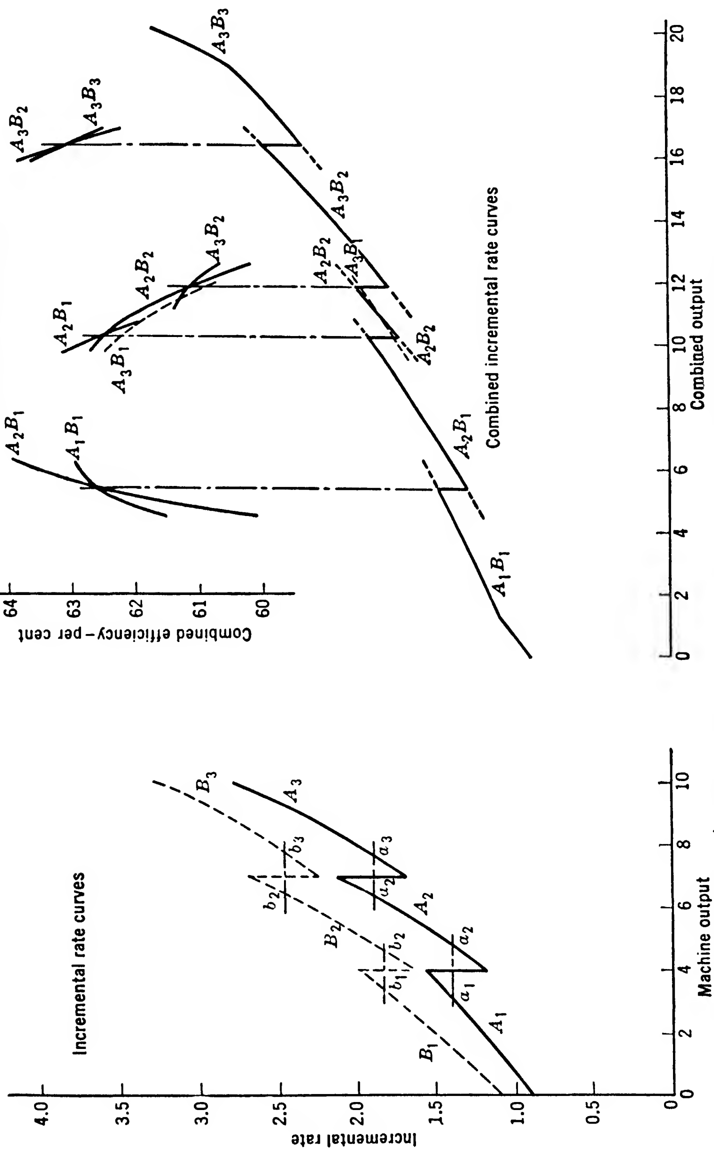


FIG. 11. Exact method of applying incremental loading to machines having Type VI (Fig. 7) input-output curves.

this figure and considering the curve for machine A, the average incremental rate represented by the line a_1a_2 can easily be established by making the area I above this line equal to the area II below the line.

Usually the equal areas can be readily determined by assuming straight-line functions for the portions of the curve involved so that each area may be considered the area of a triangle.

The significance of the average incremental rate is illustrated in Fig. 13. The discontinuity in the incremental rate curve is due to the fact that the input-output curve consists of two intersecting smooth curves.

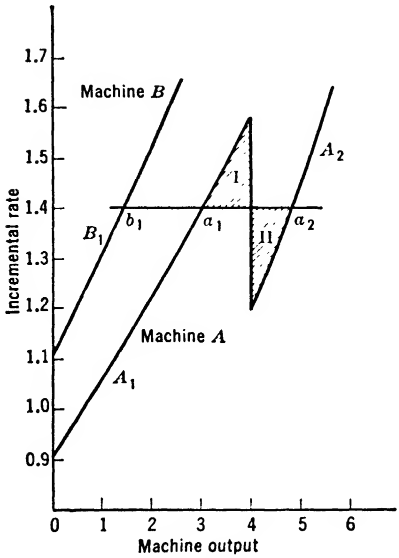


FIG. 12. Determination of average incremental rate by equal-area method.

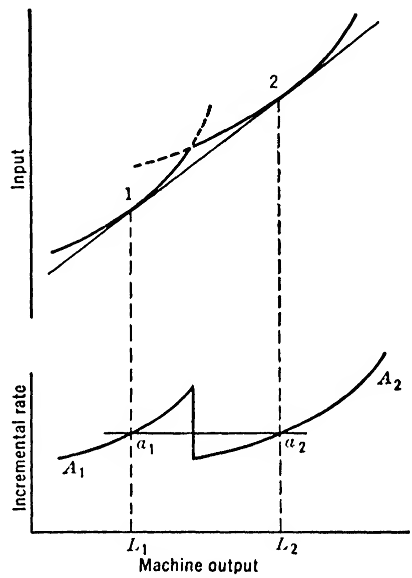


FIG. 13. Significance of average incremental rate shown in Fig. 12.

A straight line tangent to the input-output curve at points 1 and 2 will have a slope equal to the average incremental rate value represented by the line a_1a_2. This means that, if the output of the machine is increased from L_1 to L_2, the incremental output $L_2 - L_1$ can be supplied at the average incremental rate so that the corresponding incremental input will be the difference in the inputs represented by points 1 and 2.

Referring back to Fig. 12, the transition point will be the sum of the load corresponding to point b_1 and the load corresponding to the point of discontinuity on the incremental rate curve for machine A.

A comparison of the results obtained by application of the above approximate method with the exact method which employs the combined efficiency curves is tabulated below for the machines whose incremental rate curves are represented by the curves of Fig. 11.

Transition From	Transition To	Average Incremental Rate Represented By	Transition Load Approximate Method	Transition Load Exact Method
A_1B_1	A_2B_1	a_1a_2	5.45	5.40
A_2B_1	A_2B_2	b_1b_2	10.25	10.20
A_2B_2	A_3B_2	a_2a_3	11.85	11.80
A_3B_2	A_3B_3	b_2b_3	16.30	16.30

It is obvious from the above tabulation that the close agreement of the results justifies the use of the approximate method which has the additional advantage of indicating the sequence in which the various segments of the respective incremental rate curves should be combined. This is a point of considerable importance when loading more than two machines of the type under discussion. Referring to Fig. 11, it is seen that the transition from segment A_2B_1 of the combined incremental rate curve can be made either to segment A_2B_2 or to segment A_3B_1. The point of transition to the segment A_2B_2 is governed by the average incremental rate represented by the line b_1b_2 and the transition to the segment A_3B_1 by the value represented by the line a_2a_3. Since the value for b_1b_2 is less than that for a_2a_3, the proper transition is from segment A_2B_1 to segment A_2B_2. Thus the relative values of the average incremental rates represented by the lines a_1a_2, b_1b_2, a_2a_3, and b_2b_3 indicate the sequence in which the segments of the incremental rate curves for the respective machines should be combined.

Discontinuous Input-Output Curves

When the input-output curve is discontinuous, its incremental rate curve is usually discontinuous at the same points. The incremental rate may increase or decrease in passing through the points of discontinuity. Between points of discontinuity the incremental rate may always increase as the output is increased or it may decrease during some range of output.

In any event, the incremental rates alone are not sufficient to determine the division of load which will give the least input. There may be several mathematical minima to be investigated. The actual minimum combined input may not correspond to a combination of machine loads having equal incremental rates. It will then be obtained when one machine is operating at its minimum or maximum output or at an output corresponding to a point of discontinuity on its input-output curve.

The principles stated above are illustrated by the curves of Fig. 14. The loci of the points of minimum input corresponding to operation in each of the three continuous sections of the input-output curve for machine B are shown. The actual minimum combined input may be on any of the three loci. Thus, for a combined output of 4, the actual minimum

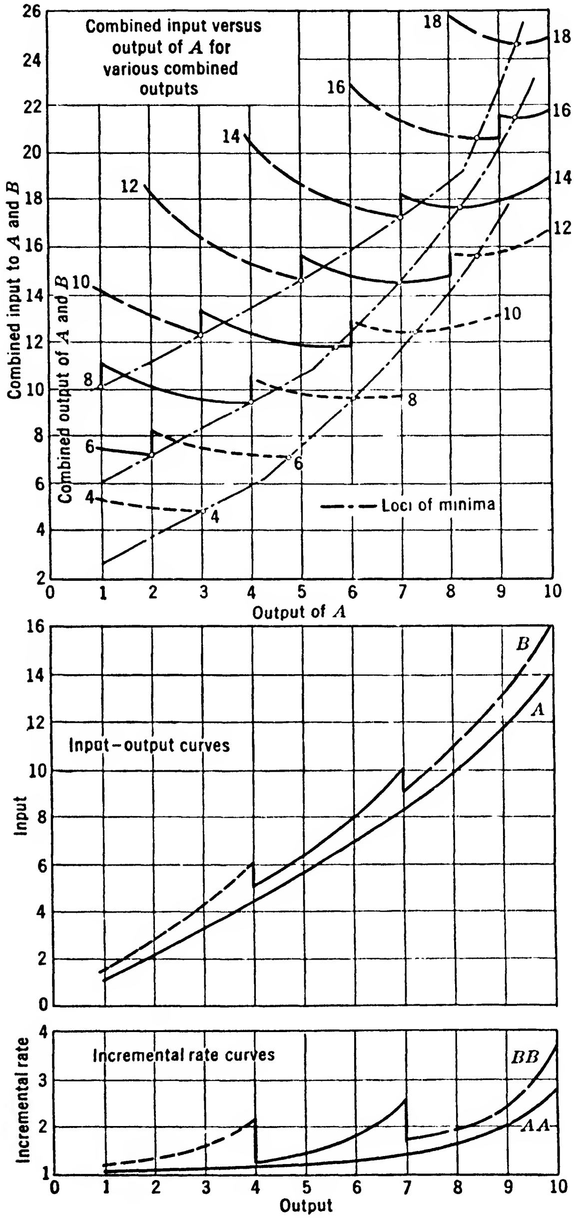


FIG. 14. Effect of load division on combined efficiency of two machines, one having a discontinuous, the other, a Type I (Fig. 7) input-output curve.

combined input occurs when machine B is operating at its minimum output and the two machines are operating at different incremental rate values. At a combined output of 8, the actual minimum combined input is obtained when machine B is operating at an output corresponding to a point of discontinuity on its input-output curve. The same would apply for a combined output of 14. At the combined outputs of 16 and 18, the actual minimum input is obtained by operating the machines at outputs corresponding to equal incremental rate values.

Summary

In this chapter consideration has been given only to the theoretical aspects of the problem of load division. Although incremental loading is universally recognized as the fundamentally correct basis for loading equipment, and is used by many large systems, little or no consideration has been given to its limitations. In this connection, curve characteristics play an important role, and therefore they have been stressed. The influence of curve characteristics on the application of incremental loading may be summarized as follows:

1. The basic performance curve of any machine is its input-output curve, which may be mathematically continuous or have mathematical points of discontinuity.
2. A continuous input-output curve may have a continuous or discontinuous incremental rate curve.
3. If the input-output curves of two or more machines are continuous, the operation of the machines in parallel at outputs corresponding to equal incremental rates will result in best overall efficiency, provided that the incremental rate curves have no decreasing values with increase in output. This rule applies to smooth continuous incremental rate curves and to discontinuous incremental rate curves whose incremental values increase at the point of discontinuity.
4. If one or more of the incremental rate curves corresponding to continuous input-output curves have decreasing incremental values with increase in output, or decreasing values at one or more points of discontinuity, the operation of the machines at outputs corresponding to equal incremental values does not necessarily result in the best overall efficiency. Under some conditions this may result in the worst overall efficiency, and incremental loading is not the sole criterion for proper load division.
5. If the input-output curves of one or more machines are discontinuous, then incremental rates alone are not sufficient to determine the proper division of load among the machines.

CHAPTER II

LOAD DIVISION IN THE BOILER ROOM

Introduction

With respect to the economic operation of a boiler plant, the fundamental problem is to obtain the minimum overall production cost at any given load or output. To attain this objective, consideration must be given to the proper allocation of load to the particular boiler units in operation, which requires the collection and use of data pertaining to the performance characteristics of the boilers. The success in solving this phase of the problem depends largely on the reliability of the available data, which in turn is influenced by the facilities available for the testing of the various boiler-plant equipment. The amount of work involved depends on the uniformity of the installation. Thus for an installation consisting of boilers of uniform design, capacity, and performance characteristics there is no problem of load division, and the amount of work involved is at a minimum, since the total boiler-room load may be equally divided among the boilers in service. On the other hand, if the installation consists of two or more groups of boilers differing in respect to design, capacity, and performance, the problem of dividing load among the groups of boilers may become very complicated and involve laborious computations unless approximate solutions are resorted to. The extent to which approximations are used should be governed by the accuracy and reliability of the basic data available. Frequently, the data on the boiler-room equipment are of such a nature that only the simplest type of calculation can be justified. Even if the data are of the highest type, consideration should be given to the use of approximate methods, if such use results in simplification and reduction in the amount of work involved, without introducing more than a negligible error in the computations.

To present a complete picture of the problem, it is necessary to discuss all the factors which influence the correct solution. However it should not be concluded from this that all the refinements should always be included in the calculations. As stated above, the extent to which they may be omitted or modified should be governed by the amount of work required if they are included, the reliability of the data used, and

the magnitude of the error which may be incurred if they are omitted or modified.

The calculations for the boiler room have two purposes in view. The first is to establish the incremental rate curves for the individual boilers so that the boiler-room load may be properly divided among them. The second is to combine these curves to represent the incremental rates for the boiler room as a whole. By properly combining these rates with the incremental rate curves for the turbine room, the incremental rates for the generating station are established so that the correct division of load among the generating stations of the system may be effected.

Methods of Loading Boilers

The problem of boiler loading is a natural result of the advancement in the art of boiler design. In the early days of the industry, it was common practice to install a large number of relatively small boilers of uniform design and performance characteristics. For this type of installation the problem of load division did not ordinarily exist, since all the boilers could be uniformly loaded for maximum overall efficiency. The output of the boilers would frequently be indicated by steam flow meters, calibrated to indicate the steaming rates of the boilers as a function of the manufacturer's rating in boiler horsepower. Thus the custom arose of operating the boilers at steaming rates corresponding to the same "per cent rating."

As larger and more efficient boilers were installed, either as replacements for or additions to existing installations, the practice of loading the boilers to maintain outputs at the same per cent rating continued for a while. This resulted in a boiler loading which was proportional to the manufacturer's rating in boiler horsepower. It was soon recognized that this practice did not result in maximum overall economy since no account was taken of the large difference in boiler efficiency between the two groups of boilers.

It was logical to assume that the more efficient boilers should generate a greater proportion of the total load. This was accomplished by base-loading the more efficient boilers until a load was reached at which the efficiencies of the two groups became equal, after which the boilers were loaded so that they operated at outputs corresponding to the same efficiencies.

With the development of the theory of incremental rate loading, a third method of load division became available.

The three methods of load division are shown in Tables II, III, and IV, which were derived from the performance curves for the two boilers shown in Fig. 15.

TABLE II

BOILER LOAD DIVISION—EQUAL RATING METHOD

Rating Per Cent	Boiler Output 10^6 Btu per Hour			Boiler Efficiency Per Cent		Boiler Input 10^6 Btu per Hour			Combined Efficiency Per Cent
	Boiler *A*	Boiler *B*	Total	Boiler *A*	Boiler *B*	Boiler *A*	Boiler *B*	Total	
100	70	90	160	80.07	88.00	87.42	102.27	189.67	84.36
120	84	108	192	80.00	87.59	105.00	123.30	228.30	84.10
140	98	126	224	79.90	87.18	122.65	144.53	267.18	83.84
160	112	144	256	79.75	86.78	140.44	165.94	306.38	83.56
180	126	162	288	79.57	86.36	158.35	187.59	345.94	83.25
200	140	180	320	79.34	85.95	176.45	209.42	385.87	82.93
220	154	198	352	79.08	85.51	194.74	231.55	426.29	82.57
240	168	216	384	78.75	85.10	213.33	253.82	467.15	82.20
260	182	234	416	78.40	84.64	232.14	276.47	508.61	81.79
280	196	252	448	78.00	84.17	251.28	299.39	550.67	81.36
300	210	270	480	77.55	83.68	270.81	322.66	593.47	80.88
320	224	288	512	77.05	83.17	290.72	346.28	637.00	80.38
340	238	306	544	76.59	82.60	310.75	370.46	681.21	79.86
357	250	321	571	76.00	82.10	328.95	390.99	719.94	79.31
380	250	342	592	76.00	81.31	328.95	420.61	749.56	78.98
400	250	360	610	76.00	80.44	328.95	447.54	776.49	78.56
422	250	380	630	76.00	79.00	328.95	481.01	809.96	77.78

TABLE III

BOILER LOAD DIVISION—EQUAL EFFICIENCY METHOD

Boiler Efficiency Per Cent		Boiler Output 10^6 Btu per Hour			Boiler Input 10^6 Btu per Hour			Combined Efficiency Per Cent
Boiler *A*	Boiler *B*	Boiler *A*	Boiler *B*	Total	Boiler *A*	Boiler *B*	Total	
80.07	88.00	70	90	160	87.42	102.27	189.67	84.36
80.07	87.31	70	120	190	87.42	137.44	224.86	84.50
80.07	86.40	70	160	230	87.42	185.19	272.61	84.37
80.07	85.46	70	200	270	87.42	234.03	321.45	83.99
80.07	84.47	70	240	310	87.42	284.12	371.54	83.44
80.07	83.39	70	280	350	87.42	335.77	423.19	82.71
80.07	82.14	70	320	390	87.42	389.58	477.00	81.76
80.07	80.07	70	363	433	87.42	453.35	540.77	80.07
79.75	79.75	112	371	483	140.44	465.20	605.64	79.75
79.50	79.50	131	374	505	164.78	470.44	635.22	79.50
79.25	79.25	145	377	522	182.97	475.71	658.68	79.25
79.00	79.00	158	380	538	200.00	481.01	681.01	79.00
78.71	79.00	170	380	550	215.98	481.01	696.99	78.91
78.18	79.00	190	380	570	243.04	481.01	724.05	78.72
77.55	79.00	210	380	590	270.81	481.01	751.82	78.48
76.82	79.00	230	380	610	299.40	481.01	780.41	78.16
76.00	79.00	250	380	630	328.95	481.01	809.96	77.78

TABLE IV

BOILER LOAD DIVISION—INCREMENTAL RATE METHOD

Incremental Rate	Boiler Output 10^6 Btu per Hour			Boiler Efficiency Per Cent		Boiler Input 10^6 Btu per Hour			Combined Efficiency Per Cent
	Boiler *A*	Boiler *B*	Total	Boiler *A*	Boiler *B*	Boiler *A*	Boiler *B*	Total	
1.165	70	90	160	80.07	88.00	87.42	102.27	189.67	84.36
1.171	70	100	170	80.07	87.76	87.42	113.95	201.37	84.42
1.182	70	120	190	80.07	87.31	87.42	137.44	224.86	84.50
1.197	70	140	210	80.07	86.86	87.42	161.18	248.60	84.47
1.208	70	160	230	80.07	86.40	87.42	185.19	272.61	84.37
1.220	70	180	250	80.07	85.95	87.42	209.42	296.84	84.22
1.235	70	200	270	80.07	85.46	87.42	234.03	321.45	83.99
1.255	70	224	294	80.07	84.89	87.42	263.84	351.26	83.70
1.260	90	230	320	79.96	84.75	112.55	271.39	383.94	83.35
1.275	110	245	355	79.79	84.35	137.87	290.46	428.33	82.88
1.290	130	261	391	79.51	83.94	163.50	310.94	474.44	82.41
1.310	150	278	428	79.16	83.46	189.50	333.09	522.59	81.90
1.338	170	296	466	78.71	82.95	215.98	356.84	572.82	81.35
1.370	190	314	504	78.18	82.36	243.04	381.25	624.29	80.73
1.405	210	330	540	77.55	81.80	270.81	403.42	674.23	80.09
1.450	230	342	572	76.82	81.31	299.40	420.61	720.01	79.44
1.500	250	353	603	76.00	80.82	328.95	436.77	765.72	78.75
1.570	250	360	610	76.00	80.44	328.95	447.54	776.49	78.56
1.800	250	380	630	76.00	79.00	328.95	481.01	809.96	77.78

In Figs. 16 and 17 are shown the loading curves and combined efficiencies resulting from the division of load by each of the three methods. Under the method of loading by equal per cent ratings, both boilers are loaded simultaneously until boiler A reaches maximum output. This range corresponds to the portion of the curve between points 1 and 2;

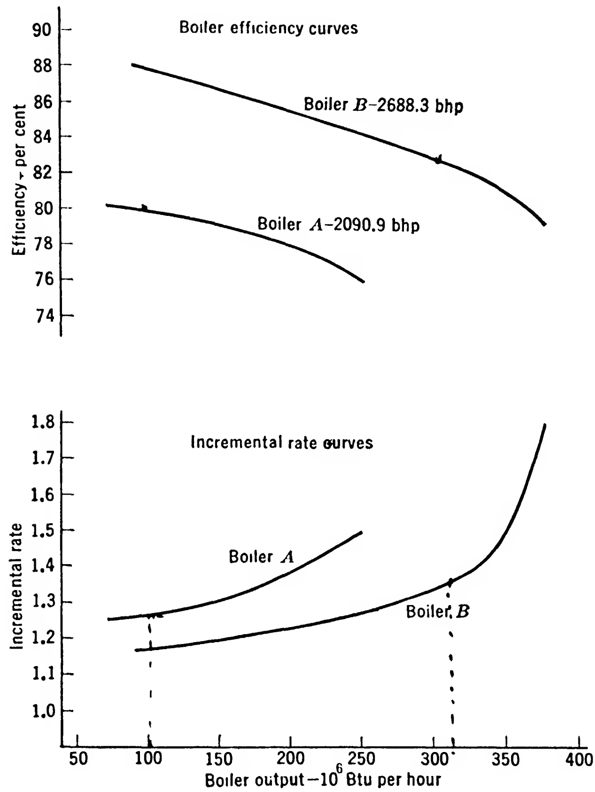


Fig. 15. Boiler performance curves.

between points 2 and 3, boiler A operates at maximum capacity while boiler B supplies the remainder of the load.

By the equal efficiency method, boiler A is held at minimum output while boiler B picks up the additional load until its efficiency is reduced to that of boiler A. This is represented by the portion of the curve between points 1 and 6. From point 6 the boilers are loaded simultaneously until boiler B reaches its maximum at point 7. From point 7 to point 3, boiler A takes the additional load.

By the incremental rate method, boiler A remains at minimum output while boiler B picks up the additional load until at point 4 its incremental

rate is equal to that of boiler A. In the range from point 4 to point 5 the boilers have equal incremental rates; from point 5, at which boiler A reaches maximum, to point 3, boiler B takes all the additional load.

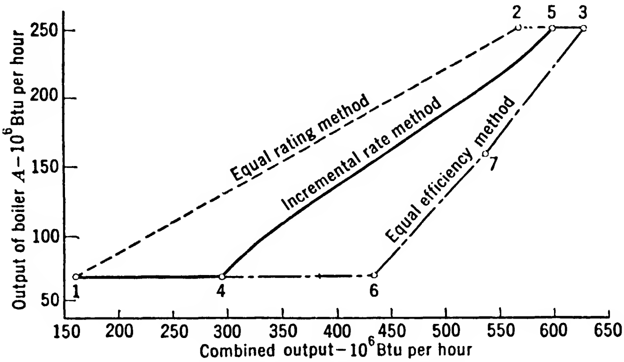


FIG. 16. Curves showing three methods of loading the boilers of Fig. 15.

The curves of Fig. 17 show that at no time is the combined efficiency based upon incremental rate loading exceeded by the efficiency obtained by either of the other two methods. In the range of output indicated between points 1 and 4 the boilers cannot be operated at outputs corresponding either to equal incremental rates or to equal efficiencies. Similarly, in the range between points 5 and 3 neither the per cent ratings

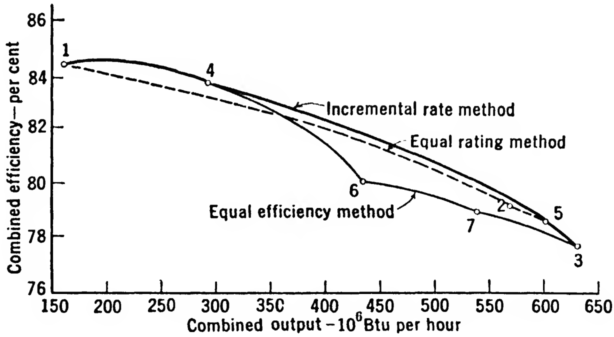


FIG. 17. Combined efficiency curves for the three methods of loading shown in Fig. 16.

nor the incremental rates corresponding to the respective outputs can be equal. Thus the coincidence of the curves in these ranges of output is due to the fact that the loading is the same as that obtained with the incremental rate method.

Boiler Efficiency Curve

For the purpose of boiler-room analysis, the boiler efficiency curve represents the most important data, and its accurate determination is essential. Some idea of the variety of boiler efficiency curves that may exist can be gathered from inspection of Fig. 18, which is typical for a large system.

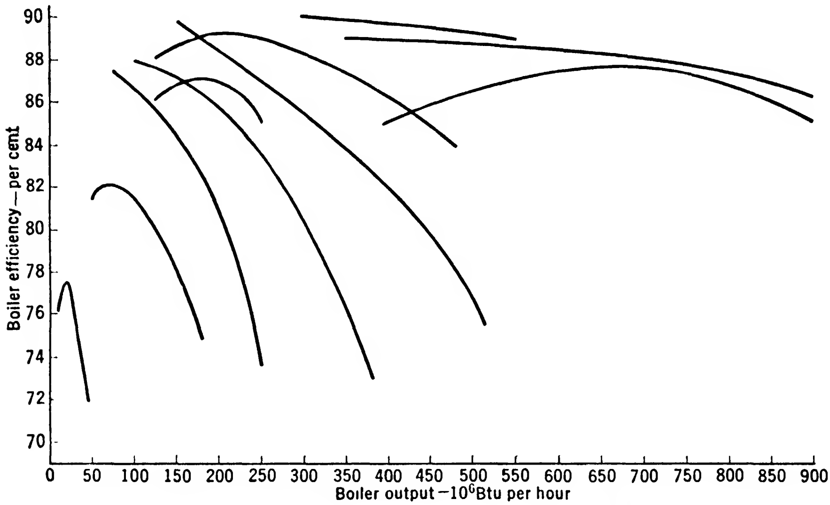


FIG. 18. Variety of boiler efficiency curves typical for a large electric system.

The boiler efficiency is generally expressed as a function of boiler output which may be indicated in three different ways:

1. As a percentage of the builder's nominal boiler-horsepower rating of the boiler.
2. By the quantity of steam delivered by the boiler, pounds per hour.
3. By the heat absorbed by the boiler, Btu per hour.

The most satisfactory measure of boiler output is the heat absorbed by the boiler unit, and it will be found that the calculations can be expedited if the boiler output is expressed in thermal units. It is recognized, however, that it may be more convenient to the boiler-room operators to have the output also expressed in one of the other forms. The heat absorbed by a boiler can be obtained from the following

$$B = Bhp \times R \times 33{,}475 \times 10^{-8} = W(H - q) \times 10^{-6} \qquad [17]$$

where B = boiler output in 10^6 Btu per hour.
Bhp = builder's nominal rating in boiler-horsepower.
R = boiler output in per cent of nominal rating.

W = steam delivered by the boiler in pounds per hour.
H = enthalpy of the steam delivered by the boiler in Btu per pound.
q = enthalpy of the feedwater supplied to the boiler in Btu per pound.
33,475 Btu per hr = 1 boiler-horsepower.

The boiler efficiency curve which should be used for load-division purposes is one which represents the performance of the boiler under actual operating conditions. This is not easily obtainable even when adequate test facilities are available. Whether the boiler efficiency curves have been judiciously selected can be determined by computing the overall boiler-room efficiency for some period of time, say one month, and comparing the computed results with the actual boiler-room performance. This check should indicate a variation of less than 1 per cent between actual and computed performance.

Some consideration might profitably be given to the various methods for establishing a boiler efficiency curve, and their relative merits.

1. *Guaranteed Performance.* The guaranteed performance of a boiler based on its design characteristics can be procured from the builder. Progress in the art of design is such that modern boilers actually do meet their guarantee performances, and when facilities for testing the boiler are not available, the use of this curve is acceptable, provided that it is first corrected to actual operating conditions.

2. *Acceptance Tests.* Shortly after a boiler is put in service it is often the practice to test the boiler for acceptance, the purpose being to determine whether the performance of the boiler meets the builder's guarantees. If the boiler is tested in accordance with prescribed test codes, the efficiency curve established by this means usually represents the best data that are possible of attainment, and indicates the ultimate efficiency at which the boiler may be expected to operate.

With increase in service hours a decrease in the efficiency is to be expected. This may be attributed to such causes as tube fouling and slagging, leaky baffles and settings, and the absence of special supervision of fire conditions. It is obvious, therefore, that some modification of the acceptance test efficiency curve is necessary, if it is to conform to the average conditions under which the boiler is expected to operate.

3. *Loss Method.* When the boiler efficiency curve cannot be established from either of the above sources, recourse can be had to the loss method, which involves the calculation of the boiler losses from a series of rather simple test observations. This method is the least desirable since it involves the estimation of the radiation and unaccounted-for

losses of the boiler. The procedure for calculating the boiler losses is readily available in any standard handbook and in the test codes for steam-generating units issued by the American Society of Mechanical Engineers.

Boiler Auxiliaries

For an academically complete solution of the problem of boiler load division it is necessary to adjust the individual boiler incremental rate curves for differences in boiler auxiliary requirements. This necessitates data which will permit the plotting of the input to the auxiliaries against the boiler output. With non-condensing non-extraction-type steam-driven auxiliaries the heat consumption chargeable against the auxiliaries is the heat input to the throttles of the auxiliary turbines less the heat recovered from the exhaust steam, which may be expressed as

$$S = W(H - h) \tag{18}$$

where S = heat consumption of or input to the steam-driven boiler auxiliaries, in Btu per hour.
W = steam supplied to the throttles of the auxiliary turbines in pounds per hour.
H = enthalpy of the steam at the throttle in Btu per pound.
h = enthalpy of the steam at the exhaust in Btu per pound.

With electrically driven auxiliaries, the input generally being expressed in kilowatts, it is first necessary to convert the kilowatt input into equivalent heat units or Btu per hour referred to the boiler outlet. The proper conversion factor for this purpose is the prevailing incremental heat rate of the main or house turbine-generator units (depending on which is the source of supply). In order to visualize the steps in the process, consider a boiler which is supplying steam to operate a turbine-generator at a given load. Assume that the boiler is provided with two sets of auxiliaries, one steam driven, and the other electrically driven. If only the steam-driven auxiliaries are used, the boiler output is greater than the input to the main turbine-generator by an amount equal to the heat input to the steam-driven auxiliaries. If the electrically driven auxiliaries are substituted for the steam-driven ones, and these are supplied by the main turbine-generator, the load on the generator would have to be increased by an amount corresponding to the consumption of the electrically driven auxiliaries, and there would have to be an increase in the input to the main turbine-generator to provide for the additional load generated.

The increase in turbine-generator load is nothing more than an "incremental output" for which the corresponding "incremental input" is

the product of the boiler auxiliary power (referred to the terminals of the main generator) and the turbine-generator incremental heat rate. The input to the electrically driven auxiliaries will then be expressed in thermal units, Btu per hour, and in a form which can be used for correcting the boiler efficiency curve.

Steam Temperature Characteristic

The economy of operation of a turbine is affected by the temperature of the steam supplied to the throttle. A decrease in temperature results in a decrease in turbine efficiency, and an increase in temperature results in an increase in efficiency. Consequently, although two boilers may be operating in parallel with equal efficiencies, if their respective steam temperatures are not the same, the steam at the lower temperature is used less efficiently in the turbine than the steam at the higher temperature.

It is evident that some adjustment is necessary for the better economy obtained in the turbine from steam at the higher temperature. This can be done by calculating the turbine-generator performance for a fixed steam temperature at the throttle and then adjusting the individual boiler efficiency curves for the variation of their respective steam temperatures from this fixed value.

In establishing the temperature characteristic of the steam as a function of the boiler output, consideration should be given to the temperature of the feedwater supplied to the boiler. Quite frequently it will be found that a boiler has been tested with feedwater at a temperature other than that which prevails under normal operating conditions. A change in feedwater temperature will change the steam temperature, and it becomes necessary to correct the steam temperature characteristic obtained under test to the temperature at which the feedwater is normally supplied to the boiler. The relation between steam temperature and feedwater temperature should preferably be established by test.

The station heat balance may be such that the feedwater is supplied to the boilers at a constant or variable temperature. A variable feedwater temperature is generally obtained when turbines are operated with a regenerative cycle involving the extraction of steam for feedwater heating; the temperature of the feedwater will then depend upon the number of turbine-generators in operation and the loads on the individual units. The effect of supplying the boilers with feedwater at a variable temperature complicates the general problem of adjusting the boiler efficiency curves for differences in steam temperature, since it requires consideration and adjustment of the turbine-generator performance. This phase of the subject is treated in greater detail in a later chapter and at this point need be considered only to the extent of bearing in mind that, if

the feedwater temperature varies, the steam temperature characteristic should be corrected to some constant feedwater temperature, preferably an average value for the station as indicated by the operating records.

Maximum Steaming Rate

The determination of the maximum steaming rate of an individual boiler presents no difficult problem. When a group of boilers is served by the same fan equipment or stack, the capacity of this equipment may not permit all the boilers to be operated simultaneously at their maximum individual steaming rates. There are many reasons why it might be undesirable to operate a boiler at its maximum steaming rate. Operation of a boiler at very high ratings generally results in excessive maintenance costs and reduces the availability of the boiler for service. It may result in steam temperatures in excess of the permissible value for the turbine. It may result in a nuisance by the excessive discharge of smoke, fly ash, or cinders from the stack.

For normal conditions, an operating limit should be established, this being a matter of judgment based on experience. By so doing, a reserve capacity is obtained which can be utilized under emergency conditions during which the economic loading of the boilers is not of primary importance.

Minimum Steaming Rate

The minimum steaming rate at which a boiler will be operated depends upon the practice in the particular station. For stoker-fired boilers, the more important of the factors which usually determine the minimum steaming rate are the ability to maintain satisfactory fuel-bed conditions, the efficiency, and the responsiveness to sudden demands which may be made upon the boilers while operating at a low rate.

For boilers burning pulverized coal, gas, or oil, the minimum rate of operation will generally depend upon the minimum burner capacity required for flame stability, and whether or not some of the burners can be cut out.

A knowledge of the minimum steaming rate for each type of boiler in the station is necessary, both for analysis to determine the proper banking sequence and for the calculation of incremental rate curves for various combinations of boilers.

Banking Losses

During the daily load cycle of a steam-generating station, there is generally a period in which some boilers are banked. The purpose of this may be to eliminate the necessity of operating boilers below their mini-

mum steaming rates or to increase the overall boiler-room efficiency. The banking practice merits careful analysis since it may disclose improper procedure with a corresponding loss in operating efficiency.

The term "banking" may be interpreted in various ways. Generally speaking, banking involves the operation of a boiler with no steam output. The resulting loss will depend on the manner of banking. If a boiler is operated with the thought in mind that it may suddenly be called upon to generate steam, then sufficient fuel is burned to maintain the boiler pressure at or slightly under line pressure. Under these conditions the boiler is said to be operating at a live bank. It may not be possible to maintain line pressure without generating a very small amount of steam, and when this condition prevails the boiler is sometimes referred to as operating at a steaming bank.

For stoker-fired boilers, in addition to maintaining steam pressure, it is necessary to maintain the fuel bed sufficiently thick to prevent the coal from being blown off the stoker when there is a sudden demand for steam. For boilers burning pulverized coal, gas, or oil, intermittent firing of one or more burners may be required to maintain the boiler at a live or steaming bank. The loss, in either type of boiler, is the fuel consumption necessary to maintain the boiler at the required condition.

If it is definitely known that the boiler will not be subject to sudden demands for steam, it may be operated at a pressure considerably less than line pressure. This is generally referred to as a dead bank condition, which is permissible for long-time banking when notice will be given in advance of the time when steam will be required from the boiler. The banking loss will obviously be less than for a live bank, since the boiler settings are permitted to cool off. Hence, when the boiler is thereafter brought up to line pressure and to a steaming condition, additional heat will be required to bring the settings back to normal operating temperatures. To obtain the average hourly loss, it is then necessary to divide the total heat input to the boiler, including the input necessary to start the boiler steaming, by the duration of the banked period.

Adjustment of Boiler Efficiency Curve to Operating Conditions

Before using the boiler test curve it is desirable that it be checked against the actual performance of the boiler under average conditions of cleanliness. Operating records will generally disclose the number of boiler service hours between periodic outages for overhaul and cleaning, and the boiler may be considered to be at average cleanliness after it has been in operation for half the usual period between cleanings.

The check is made by comparing the average boiler efficiency obtained over a given period of operation, with the "bogey" average effi-

TABLE V

CALCULATION TO CHECK COMPUTED EFFICIENCY AGAINST ACTUAL EFFICIENCY

Time	Duration Hours	Steam Flow Lb per Hr	Temperature, Deg F		Enthalpy, Btu per Lb		Boiler Output, 10^6 Btu		Boiler Efficiency from Test Curve Per Cent	Total Boiler Input 10^6 Btu
			Steam	Feedwater	Steam *	Feedwater	Per Hour	Total		
12– 8 A.M.	8	100,000	670	200	1,345.3	168.0	117.7	941.6	89.4	1,053.2
8– 9	1	150,000	700	216	1,361.9	184.1	176.7	176.7	89.0	198.5
9–12	3	200,000	710	232	1,367.3	200.2	233.4	700.2	84.1	832.6
12– 1 P.M.	1	175,000	705	224	1,364.6	192.2	205.2	205.2	87.0	235.9
1– 4	3	200,000	710	232	1,367.3	200.2	233.4	700.2	84.1	832.6
4– 6	2	250,000	715	245	1,370.1	213.4	289.2	578.4	76.6	755.1
6– 8	2	225,000	710	238	1,367.3	206.3	261.2	522.4	80.6	648.1
8– 9	1	175,000	705	224	1,364.6	192.2	205.2	205.2	87.0	235.9
9–10	1	150,000	700	216	1,361.9	184.1	176.7	176.7	89.0	198.5
10–11	1	125,000	690	208	1,356.4	176.0	147.6	147.6	90.0	164.0
11–12	1	100,000	670	200	1,345.3	168.0	117.7	117.7	89.4	131.7
Totals	24							4,471.9		5,286.1

* At 415 psi abs.

Computed overall efficiency for 24-hour period $\frac{4471.9}{5286.1} = 84.6\%$

Actual performance for 24-hour period

Coal burned (from station records) 393,411 Lb

Calorific value of coal 14,100 Btu per Lb

Boiler input (393,411 × 14,100) 5547.1 × 10^6 Btu

Overall efficiency $\left(\frac{4471.9}{5547.1}\right)$ 80.6%

Operating factor $\frac{80.6}{84.6} = 0.953$

ciency derived by integrating the test values over a boiler-load-duration curve for the period. The load-duration curve can be easily prepared if the boiler is equipped with reliable steam flow or feedwater meters, and if the steam pressures and temperatures and the feedwater temperatures are made available either by measurement or from recording meters. The input to the boiler would be determined by the quantity of coal burned and its calorific value. A sample calculation is shown in Table V.

If adjustment of the test curve is indicated by reason of the procedure outlined above, the problem to be met becomes the determination of how the adjustment should be made. One method is to multiply the efficiencies of the test curve by the constant operating factor derived in the manner indicated in Table V. The principal objection to this method is the fact that the ratio of the operating efficiency to the test efficiency is probably not a constant but varies with the boiler output, being a maximum at minimum output and a minimum at maximum output. Since the operating characteristic of most boilers is such that the efficiency is highest at or near its minimum operating output and lowest at its maximum rating, it follows that the application of a constant factor to the test curve will result in an adjusted curve with the greatest reduction in efficiency at the output corresponding to the highest value of the test efficiency, and the least reduction in efficiency at the output corresponding to the lowest test efficiency.

Actually, general experience indicates that the effect of boiler fouling is to cause the least reduction in efficiency at or near minimum rating and the largest reduction at maximum rating, a condition which is the reverse of that obtained when a constant operating factor is applied. The conclusion is obvious, therefore, that the application of a constant operating factor should be resorted to only when no other means of adjustment are available.

The best method for determining the boiler efficiency curve for average cleanliness is by test, which should be simplified to whatever extent facilities will permit. Utilization of available recording instruments to determine steam flow, pressure, temperatures, and quantity of fuel burned is recommended. The boiler should be operated by the attendant normally assigned to it, without giving it any special attention, and at a sufficient number of constant loads to permit the determination of the efficiency curve.

A comparison of both methods is shown in Fig. 19. Curve A shows the efficiency under test conditions; curve B, the operating efficiency at average boiler cleanliness determined by applying a constant factor to the test efficiency; and curve C shows the operating efficiency determined by test. The corresponding boiler incremental rates are shown by curves

AA, *BB*, and *CC*. Inspection of the incremental rate curves indicates that the use of a constant operating factor will give incremental rate values which are lower than those derived from the actual operating efficiency curve, and hence will result in incorrect load division in the boiler room and in station incremental rate curves which do not correctly define the performance of the station.

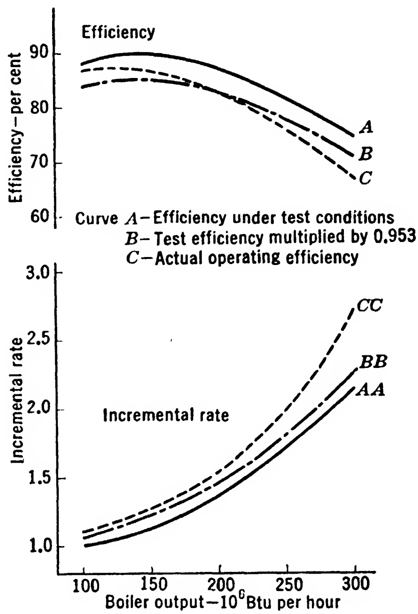


FIG. 19. Effect of a constant boiler efficiency correction factor on the incremental rate curve.

With respect to an individual boiler, the operating efficiency curve will be pessimistic during the early periods of service following the overhaul of the boiler, and optimistic in the period preceding its overhaul. With respect to a group of boilers, however, the diversity in the cleanliness of the individual boilers will make the cleanliness of the group as a whole close to the average to which the test efficiency curve of the individual boilers have been adjusted. This follows from the fact that it is the usual practice to stagger the outage of boilers for overhaul in a plant, so that the individual boilers are in different stages of cleanliness at any given time.

Adjustment for Steam Temperature

The difficulties encountered in adjusting the boiler efficiency curve for variation in steam temperature can best be understood by reference to the simplest type of installation. Consider a single turbine-generator supplied with steam from a single boiler, and let it be assumed that the auxiliaries for the boiler and turbine are electrically driven and that the feedwater to the boiler is at a constant temperature. Under these conditions, the boiler output is equal to the turbine input, and the combined efficiency of the boiler and turbine-generator may be expressed as

$$E = E_b \times E_t \qquad [19]$$

where E = combined efficiency of the boiler and turbine-generator.

E_b = efficiency of the boiler.

E_t = efficiency of the turbine-generator.

The boiler efficiency E_b is readily obtained, since it is a function of the boiler output. The turbine-generator efficiency E_t, however, cannot be directly determined since, for a given load on the generator, the required output of the boiler will depend on the temperature of the steam corresponding to the boiler output. It therefore becomes necessary to establish the relation between the turbine-generator efficiency, the boiler output, and the steam temperature. This can be done by plotting the turbine-generator efficiency as a function of the boiler output in a series of curves for different steam temperatures. The steam temperature for a given boiler output being known, the corresponding turbine-generator efficiency is then obtained directly or by interpolation from the curves. The calculations involved become too cumbersome for practical application to more than one turbine-generator and one boiler. For example, when two or more boilers with different steam-temperature characteristics operate in parallel, the temperature of the steam delivered to the turbine room through a common header will depend upon the outputs of the individual boilers and the steam temperatures at the respective boiler outputs. This introduces an additional item to be calculated. Furthermore, the efficiency of a turbine-generator is influenced by the vacuum at the exhaust of the turbine as well as by the temperature of the steam supplied to its throttle. This greatly increases the number of curves that must be established for the turbine-generator under the procedure described above.

To reduce the calculations to a minimum, the equivalent efficiency or equivalent heat rate of the turbine-generator should be established, corrected to a fixed or standard steam temperature. Usually, a series of curves is so established for which the vacuum at the turbine exhaust is made the parameter. (This phase is discussed in greater detail in the next chapter.) With respect to equation 19 this operation changes the value of E_t so that it becomes necessary to change the boiler efficiency E_b in such a manner that the combined efficiency E will remain constant and unaffected.

To illustrate the method for adjusting the boiler efficiency to the standard temperature, for a single boiler and single turbine-generator, let

T_s = standard steam temperature to which the turbine and boiler performances are corrected, degrees Fahrenheit.

T_a = actual steam temperature at the turbine throttle, degrees Fahrenheit.

W_s = turbine steam consumption at standard temperature corresponding to some given load, pounds per hour.

W_a = turbine steam consumption at actual steam temperature and same given load, pounds per hour.

I_a = actual boiler input, Btu per hour.
B_a = actual boiler output required when steam is at temperature T_a, Btu per hour.
B_s = equivalent boiler output which would be required if steam were at the temperature T_s, Btu per hour.
H_s = enthalpy of the steam at temperature T_s, Btu per pound.
H_a = enthalpy of the steam at temperature T_a, Btu per pound.
q = enthalpy of feedwater into boiler, Btu per pound.
c = turbine steam consumption correction factor for variation of superheat at turbine throttle.[1]

Then

$$B_s = W_s(H_s - q) \qquad [20]$$

$$B_a = W_a(H_a - q) \qquad [21]$$

But

$$W_a = W_s[1 + c(T_s - T_a)] \qquad [22]$$

Therefore

$$B_a = W_s[1 + c(T_s - T_a)](H_a - q) \qquad [23]$$

and

$$\frac{B_s}{B_a} = \frac{(H_s - q)}{[1 + c(T_s - T_a)](H_a - q)} \qquad [24]$$

or

$$\frac{B_s}{B_a} = \frac{(H_s - q)}{C(H_a - q)} \qquad [25]$$

where

$$C = 1 + c(T_s - T_a)$$

Equation 25 indicates the ratio of the boiler output at standard steam temperature to the actual output.

Let O_t = output of the generator expressed in thermal units, Btu per hour; then, under actual conditions, $E_t = \frac{O_t}{B_a}$; $E_b = \frac{B_a}{I_a}$; and

$$E = E_t \times E_b = \frac{O_t}{B_a} \times \frac{B_a}{I_a} = \frac{O_t}{I_a} \qquad [26]$$

Equation 26 expresses the combined efficiency as the ratio of the generator output to the boiler input.

[1] The correction factor is assumed to correspond to constant load. In modern practice separate corrections are made to the steam flow and steam rate.

Letting E'_b and E'_t = the boiler and turbine-generator efficiencies, respectively, adjusted to the standard steam temperature, T_s, then

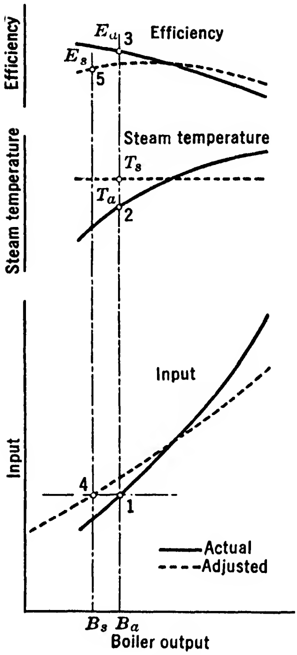


FIG. 20. Adjustment of boiler efficiency and input-output curves to a standard steam temperature.

$$E'_t = \frac{O_t}{B_s} \qquad [27]$$

and

$$E = E_t \times E_b = E'_t \times E'_b \qquad [28]$$

Hence

$$\frac{E'_b}{E_b} = \frac{E_t}{E'_t} = \frac{O_t}{B_a} \div \frac{O_t}{B_s} \qquad [29]$$

or

$$\frac{E'_b}{E_b} = \frac{B_s}{B_a} = \frac{(H_s - q)}{C(H_a - q)} \qquad [30]$$

and

$$E'_b = E_b \times \frac{B_s}{B_a} = \frac{B_a}{I_a} \times \frac{B_s}{B_a} = \frac{B_s}{I_a} \qquad [31]$$

The application of the adjustment indicated by equations 25, 30, and 31 is graphically illustrated in Fig. 20. For a given load on the generator, the actual boiler output B_a is represented by point 1 on the input-output curve. The corresponding steam temperature T_a and boiler efficiency E_b are indicated by points 2 and 3, respectively. At standard temperature T_s the required boiler output B_s for the given generator load is calculated from equation 25, and the corresponding efficiency E'_b from equation 31. The actual input I_a remains unchanged, and it is plotted against the adjusted boiler output B_s, thereby fixing point 4 on the adjusted input-output curve. The adjusted efficiency E'_b is also plotted against the adjusted output B_s, fixing point 5 on the adjusted efficiency curve. An actual calculation by this procedure is shown in Table VI.

The adjusted values of output and efficiency are used as though they were the actual values. This condition must prevail if the overall performance of the boiler and turbine-generator is to remain unaffected by the adjustment of the turbine-generator and boiler performance to the standard temperature T_s.

Boiler-loading schedules are usually expressed in terms of steam flow. This requires the conversion of the adjusted output expressed in thermal units to the corresponding actual steam flow. The tabular data necessary

TABLE VI

ADJUSTMENT OF BOILER EFFICIENCY TO STANDARD STEAM TEMPERATURE

B_a 10^6 Btu per Hour	T_a Deg F	$T_s - T_a$ Deg F	C	H_a Btu per Lb	$H_a - q$ Btu per Lb	$\frac{H_s - q}{H_a - q}$	B_s/B_a	B_s 10^6 Btu per Hour	E_a Per Cent	E_s Per Cent	W_a 1,000 Lb per Hour
100	617	83	1.083	1,325.4	1,147.4	1.0383	0.9587	95.9	87.00	83.41	87.15
110	636	64	1.064	1,335.6	1,157.6	1.0292	0.9673	106.4	87.48	84.62	95.02
120	653	47	1.047	1,344.6	1,166.6	1.0213	0.9755	117.1	87.90	85.75	102.86
130	666	34	1.034	1,351.5	1,173.5	1.0153	0.9819	127.6	88.24	86.64	110.78
140	678	22	1.022	1,357.8	1,179.8	1.0098	0.9881	138.3	88.51	87.46	118.66
150	688	12	1.012	1,363.1	1,185.1	1.0053	0.9934	149.0	88.73	88.14	126.57
160	696	4	1.004	1,367.3	1,189.3	1.0018	0.9978	159.6	88.88	88.68	134.53
170	703	− 3	0.997	1,371.0	1,193.0	0.9987	1.0017	170.3	88.92	89.07	142.50
180	709	− 9	0.991	1,373.1	1,195.1	0.9969	1.0060	181.1	88.90	89.43	150.62
190	714	−14	0.986	1,376.7	1,198.7	0.9939	1.0080	191.5	88.80	89.51	158.51
200	718	−18	0.982	1,378.8	1,200.8	0.9922	1.0104	202.1	88.53	89.45	166.56
210	721	−21	0.979	1,380.3	1,202.3	0.9909	1.0122	212.6	88.18	89.26	174.67
220	724	−24	0.976	1,381.9	1,203.9	0.9896	1.0139	223.1	87.60	88.82	182.74
230	726	−26	0.974	1,383.0	1,205.0	0.9887	1.0151	233.5	86.83	88.14	190.87
240	728	−28	0.972	1,384.0	1,206.0	0.9879	1.0164	243.9	85.80	87.21	199.00
250	730	−30	0.970	1,385.1	1,207.1	0.9870	1.0175	254.4	84.30	85.78	207.11

T_s = 700° F
Pressure = 280 psi abs
q = 178 Btu per Lb
H_s = 1369.4 Btu per Lb
$H_s - q$ = 1191.4 Btu per Lb

Steam rate correction (c) = 0.001 per deg F

$$C = 1 + c(T_s - T_a)$$

$$\frac{B_s}{B_a} = \frac{E_s}{E_a} = \frac{1}{C}\left(\frac{H_s - q}{H_a - q}\right)$$

for computing the adjusted efficiency curve, as shown in Table VI, are sufficient to establish the actual steam flow W_a by means of equation 21. The actual steam flow corresponding to the adjusted outputs can be graphically shown by means of an auxiliary scale or as a curve, both methods being illustrated in Fig. 23.

The correction for steam-temperature variation is frequently applied to the turbine-generator heat rates rather than to the steam rates. By adding the following notation to that above, the relation between B_s and B_a is derived as follows:

Let HR_s = turbine-generator heat rate at the standard steam temperature T_s, Btu per kilowatt-hour.

HR_a = turbine-generator heat rate at the actual steam temperature T_a, Btu per kilowatt-hour.

L = generator load, kilowatts.

k = correction factor applied to the heat rate for variation in the superheat of the steam at the turbine throttle.

Then

$$HR_a = \frac{W_a(H_a - q)}{L} \quad [32]$$

and

$$HR_s = \frac{W_s(H_s - q)}{L} \quad [33]$$

But

$$HR_a = HR_s[1 + k[T_s - T_a)] = K \times HR_s \quad [34]$$

where

$$K = 1 + k(T_s - T_a)$$

Therefore

$$\frac{HR_s}{HR_a} = \frac{1}{K} = \frac{W_s(H_s - q)}{L} \div \frac{W_a(H_a - q)}{L} = \frac{W_s(H_s - q)}{W_a(H_a - q)} \quad [35]$$

But

$$B_s = W_s(H_s - q) \qquad \text{and} \qquad B_a = W_a(H_a - q) \quad [36]$$

Therefore

$$\frac{B_s}{B_a} = \frac{1}{K} = \frac{E_b'}{E_b} = \frac{1}{[1 + k(T_s - T_a)]} \quad [37]$$

In the discussion so far, it has been assumed that all the station auxiliaries are electrically driven so that in substance the output of the boiler might be considered equal to the input to the turbine-generator unit. When the auxiliaries are steam driven, the above assumption no

longer prevails, and adjustment of the boiler outputs to the standard temperature is then derived as follows:

Let W_a and W_s = the steam delivered by the boiler at the actual and standard temperatures, T_a and T_s, respectively, pounds per hour.

W_{ta} and W_{ts} = the steam to the main turbine throttle at the actual and standard temperatures, T_a and T_s, respectively, pounds per hour.

A_a and A_s = the steam to the auxiliary turbine throttles at the actual and standard temperatures, T_a and T_s, respectively, pounds per hour.

c and c' = the correction factors applied to the steam rates for variation in superheat of the steam to the throttles of the main and auxiliary turbines, respectively.

Then

$$W_a = W_{ta} + A_a \qquad \text{and} \qquad W_s = W_{ts} + A_s \tag{38}$$

and

$$B_a = W_a(H_a - q) = (W_{ta} + A_a)(H_a - q) \tag{39}$$

also

$$B_s = W_s(H_s - q) = (W_{ts} + A_s)(H_s - q) \tag{40}$$

But

$$W_{ta} = W_{ts}[1 + c(T_s - T_a)] \qquad \text{or} \qquad W_{ts} = \frac{W_{ta}}{[1 + c(T_s - T_a)]} \tag{41}$$

and

$$A_a = A_s[1 + c'(T_s - T_a)] \qquad \text{or} \qquad A_s = \frac{A_a}{[1 + c'(T_s - T_a)]} \tag{42}$$

Therefore

$$\frac{B_s}{B_a} = \frac{\left[\dfrac{W_{ta}}{1 + c(T_s - T_a)} + \dfrac{A_a}{1 + c'(T_s - T_a)}\right]}{W_{ta} + A_a}\left(\frac{H_s - q}{H_a - q}\right) \tag{43}$$

Equation 43 can be applied when the total station auxiliary steam consumption A_a is known. When the load and combination of boilers and turbine-generator units in service are known, it is a relatively simple matter to establish the total auxiliary steam consumption. Since it is desired to calculate the performance in the boiler room independently of the performance in the turbine room, no simple method is available by

which to establish the value of A_a, because the auxiliary steam consumption in the turbine room is independent of that in the boiler room. For this reason, the use of equation 43 is limited to the case where the overall performance of the station is to be calculated for a given combination of boilers and turbine-generators.

No material error is introduced in the calculations if the value of c' be assumed to be equal to that of c. This follows from the fact that the quantity of steam used by the station auxiliaries is generally small in comparison with the total quantity generated by the boilers. Under this assumption, equation 43 reduces to equation 24, permitting the independent calculation of the boiler-room performance.

Adjustment for Auxiliary Power Consumption

When the boiler-room auxiliaries are steam driven, the effect of their steam consumption is to reduce the amount of steam available for use in the turbine room. At a given boiler output the effect is to reduce the load generated by the main turbine-generators.

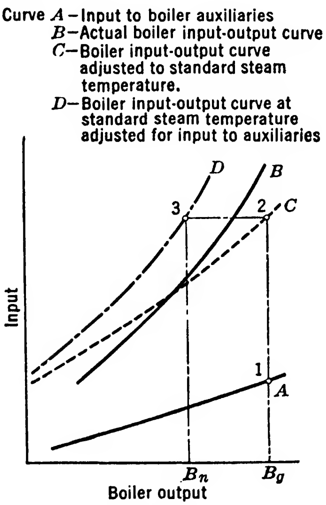


FIG. 21. Adjustment of boiler input-output curve for auxiliary power consumption.

If the boiler auxiliaries are electrically driven, the total boiler output is available to the throttles of the main turbine-generators, and for the same boiler output there is an increase in the gross generated load. The effect of the boiler auxiliaries is to decrease the load available at the station bus for transmission and distribution.

Hence, regardless of the type of auxiliary drive, the adjustment of the boiler input-output curve should consist of a reduction in boiler output by an amount equal to the thermal input to its auxiliaries. The method of doing this, assuming that the input to the boiler auxiliaries is available in terms of thermal units and as a function of the boiler output, is illustrated in Fig. 21. At any gross boiler output B_g the input to the boiler auxiliaries is obtained from the auxiliary power curve at point 1. The input to the auxiliaries is subtracted from the gross boiler output at point 2 on the boiler input-output curve (which has been adjusted to the standard steam temperature), locating point 3. The difference between the outputs at point 2 and point 3 is equal to the input to the boiler auxiliaries, represented by point 1. The output corresponding to point 3

therefore represents the net boiler output B_n. By repeating this process for several values of the gross boiler output, enough points are obtained to permit the plotting of the boiler input-output curve adjusted to the standard steam temperature and for the power consumption of the boiler auxiliaries.

Incremental Rate Curve for an Individual Boiler

The calculation of the individual boiler incremental rate curve incorporating the corrections for variation of steam temperature from the standard value and for its auxiliary power consumption is a relatively simple matter. If the boiler efficiency curve is available, the boiler incremental rate curve can be calculated without plotting the corresponding

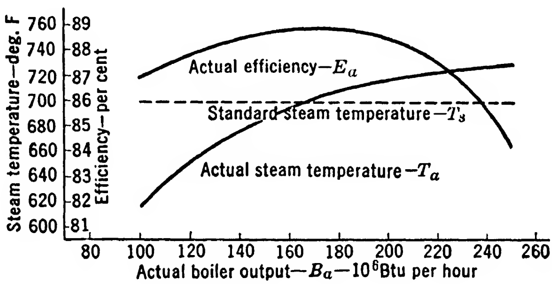


FIG. 22. Actual boiler performance curves.

input-output curve. The step-by-step procedure is illustrated in Tables VI and VII for the actual boiler performance curves shown in Fig. 22.

The first step is to correct the boiler efficiency curve to the standard steam temperature as shown in Table VI.

For the second step, the efficiency curve adjusted to the standard temperature is plotted. This curve is shown in Fig. 23, having been obtained by plotting the values of E_s against the boiler output B_s.

The final step is the calculation of the boiler incremental rates, incorporating the correction for the boiler auxiliary power consumption. This calculation is shown in Table VII.

In column 1 of this table, boiler outputs are selected so that the successive differences in output (the incremental outputs) are equal.

In column 2 are recorded the boiler efficiencies corresponding to the outputs of column 1. These values are read from the boiler efficiency curve adjusted to the standard temperature, shown in Fig. 23.

The values of column 3 are obviously obtained by dividing the values of column 1 by the corresponding values of column 2.

Column 4 indicates the incremental boiler inputs, obtained as the differences between successive values of column 3.

The auxiliary power consumption corresponding to the boiler outputs of column 1 are shown in column 5. These are read from the auxiliary power consumption curve of Fig. 23, which has been corrected to the standard temperature T_s in accordance with equation 42.

The net boiler outputs are tabulated in column 6, obtained as the differences between the corresponding values of columns 1 and 5.

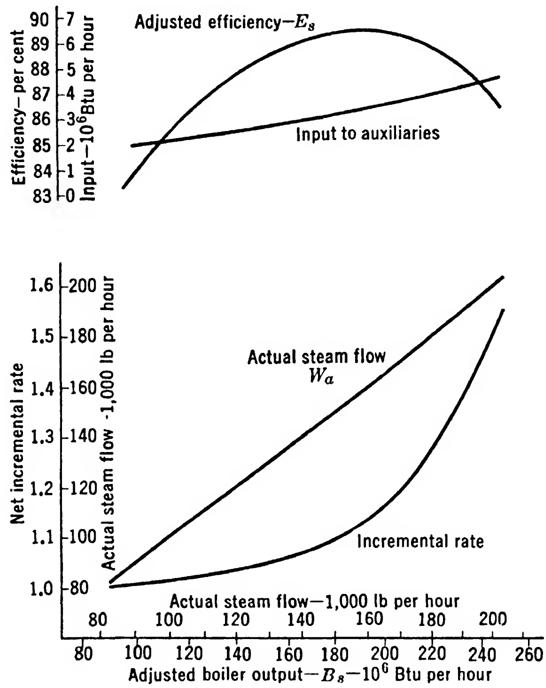


FIG. 23. Adjusted boiler performance curves.

The net boiler incremental outputs are shown in column 7. These are the differences between successive values of column 6.

In column 8 are tabulated the net boiler incremental rates, which are obtained by dividing the values of column 4 by the corresponding values of column 7.

The net incremental rates are plotted against the adjusted gross boiler outputs, the values of which are indicated in column 9. The incremental rates are, in fact, averages for the intervals of output represented by the differences between the successive values of boiler output tabulated in column 1. The outputs against which the incremental rates are plotted, indicated in column 9, are therefore the arithmetical averages of the successive values of column 1.

TABLE VII

Calculation of Boiler Incremental Rate Curve

Gross Boiler Output * (B_g) 10^6 Btu per Hour	Boiler Efficiency * (E_s) Per Cent	Boiler Input (I) 10^6 Btu per Hour	Incremental Boiler Input (ΔI) 10^6 Btu per Hour	Auxiliary Power Consumption 10^6 Btu per Hour	Net Boiler Output (B_n) 10^6 Btu per Hour	Incremental Net Boiler Output (ΔB_n) 10^6 Btu per Hour	Net Incremental Rate (R_b)	Output against Which R_b is Plotted 10^6 Btu per Hour
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100	83.80	119.33		1.98	98.02			
			9.98			9.86	1.012	105
110	85.07	129.31		2.12	107.88			
			10.05			9.86	1.019	115
120	86.11	139.36		2.26	117.74			
			10.08			9.86	1.022	125
130	86.99	149.44		2.40	127.60			
			10.20			9.86	1.034	135
140	87.70	159.64		2.54	137.46			
			10.27			9.86	1.042	145
150	88.28	169.91		2.68	147.32			
			10.37			9.86	1.052	155
160	88.75	180.28		2.82	157.18			
			10.50			9.86	1.065	165
170	89.11	190.78		2.96	167.04			
			10.63			9.85	1.079	175
180	89.37	201.41		3.11	176.89			
			10.86			9.81	1.107	185
190	89.51	212.27		3.30	186.70			
			11.17			9.80	1.140	195
200	89.51	223.44		3.50	196.50			
			11.59			9.80	1.183	205
210	89.35	235.03		3.70	206.30			
			12.16			9.80	1.241	215
220	89.00	247.19		3.90	216.10			
			12.81			9.77	1.311	225
230	88.46	260.00		4.13	225.87			
			13.69			9.73	1.407	235
240	87.69	273.69		4.40	235.60			
			15.03			9.70	1.500	245
250	86.59	288.72		4.70	245.30			

* At standard temperature (T_s).

The incremental rate curve plotted from columns 8 and 9 of Table VII is shown in Fig. 23. It is to be noted that the incremental rate values at the minimum and maximum boiler outputs are obtained by extrapolating the curve beyond the plotted values.

The curves of Fig. 23 represent the final data which are to be used for load-division purposes. When similar curves are established for all groups of boilers in the station, it then becomes a relatively simple matter to establish an overall incremental rate curve for any combination of boilers desired. It is to be noted that the relation between actual steam flow and adjusted boiler output B_s is shown in two ways. Either may be used to facilitate the preparation of loading schedules indicating boiler output as a function of steam flow.

Incremental Rate Curve for a Group of Boilers

For a group of similar boilers having identical performance characteristics, the incremental rate curve for any number of boilers is derived

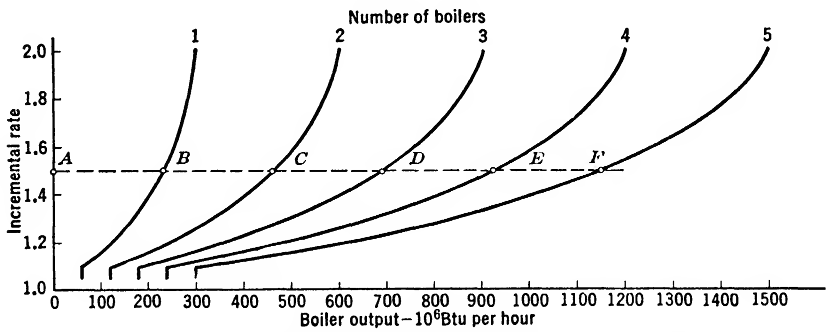


FIG. 24. Incremental rate curves for groups of similar boilers.

from the incremental rate curve for a single boiler by plotting the values of its incremental rate against the products of the corresponding output values and the number of boilers in the group. This is illustrated in Fig. 24. Thus, at an incremental rate of 1.5, the output of one boiler is at B, for 2 boilers at C, for 3 boilers at D, etc., so that $AB = BC = CD = DE = EF$. By repeating the process for other incremental rate values, enough points are established to permit drawing the curves for the respective groups of boilers. The same results can be established in tabular form, as illustrated by Table VIII. The values in the first two columns of this table are obtained from the incremental rate curve for an individual boiler. The output for any number of similar boilers is obtained by multiplying the output of one boiler by the number of boilers in the group. When the outputs for the respective number of

boilers are plotted against the incremental rates, a series of curves is obtained as shown in Fig. 24.

When the boiler installation consists of two or more groups of boilers having dissimilar characteristics, there are two problems to be met. The

TABLE VIII

CALCULATION OF INCREMENTAL RATE CURVES FOR GROUPS OF SIMILAR BOILERS

Incremental Rate	Output, 10^6 Btu per Hour				
	Number of Boilers in Group				
	1	2	3	4	5
1.1	60	120	180	240	300
1.2	120	240	360	480	600
1.3	165	330	495	660	825
1.4	200	400	600	800	1,000
1.5	230	460	690	920	1,150
1.6	252	504	756	1,008	1,260
1.7	270	540	810	1,080	1,350
1.8	284	568	852	1,136	1,420
1.9	294	588	882	1,176	1,470
2.0	300	600	900	1,200	1,500

first relates to the division of load between the respective groups of boilers. When this load division has been determined, then the load assigned to each group is divided equally among the steaming boilers in the respective groups. The second problem is to establish the incremental rate curve for any combination of boilers in service, since this curve will be required to establish the overall incremental heat rate curve for the station.

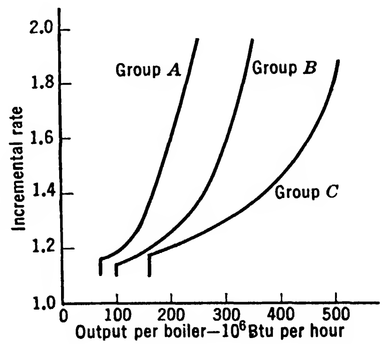


FIG. 25. Incremental rate curves for single dissimilar boilers.

The solution of both problems is primarily a function of load division among single boilers that are typical for each group. To illustrate, consider an installation consisting of twenty boilers arranged in five rows with four boilers in each row. With respect to performance characteristics, the boilers are divided into three groups, four in the first group and eight in each of the second and third groups. The incremental rate curves for one boiler in each group are shown in Fig. 25.

TABLE IX

CALCULATION OF INCREMENTAL RATE CURVE FOR GROUPS OF DISSIMILAR BOILERS

Incremental Rate	Output, 10^6 Btu per Hour						Total 15 Boilers
	Per Boiler			Per Group			
	A	*B*	*C*	*A* 3 Boilers	*B* 6 Boilers	*C* 6 Boilers	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.140	70	100	160	210	600	960	1,770
1.150	70	110	160	210	660	960	1,830
1.160	70	120	160	210	720	960	1,890
1.170	75	132	160	225	792	960	1,977
1.175	90	140	160	270	840	960	2,070
1.200	110	160	192	330	960	1,152	2,442
1.250	132	196	252	396	1,176	1,512	3,084
1.300	146	222	300	438	1,332	1,800	3,570
1.350	158	243	340	474	1,458	2,040	3,972
1.400	168	260	370	504	1,560	2,220	4,284
1.450	178	274	396	534	1,644	2,376	4,554
1.500	186	286	418	558	1,716	2,508	4,782
1.600	202	305	453	606	1,830	2,718	5,154
1.700	217	322	480	651	1,932	2,880	5,463
1.800	231	336	498	693	2,016	2,988	5,697
1.880	243	346	510	729	2,076	3,060	5,865
1.900	246	348	510	738	2,088	3,060	5,886
1.940	252	353	510	756	2,118	3,060	5,934
1.965	254	355	510	762	2,130	3,060	5,952

The load division among single boilers, one for each group, and for a combination consisting of three boilers in one group and six boilers in each of the remaining groups, is shown in Table IX. Referring to this

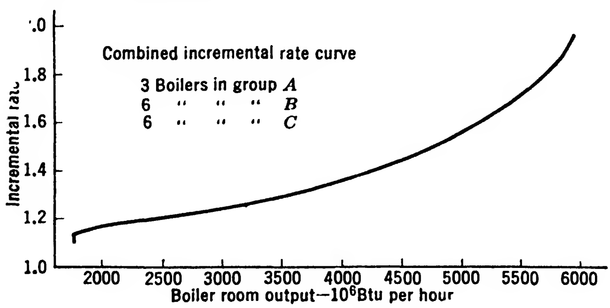


FIG. 26. Incremental rate curve for a combination of 15 boilers of Fig. 25.

table, columns 1, 2, and 3 indicate the outputs for one boiler in each group. Columns 4, 5, and 6 are the respective group loadings for the

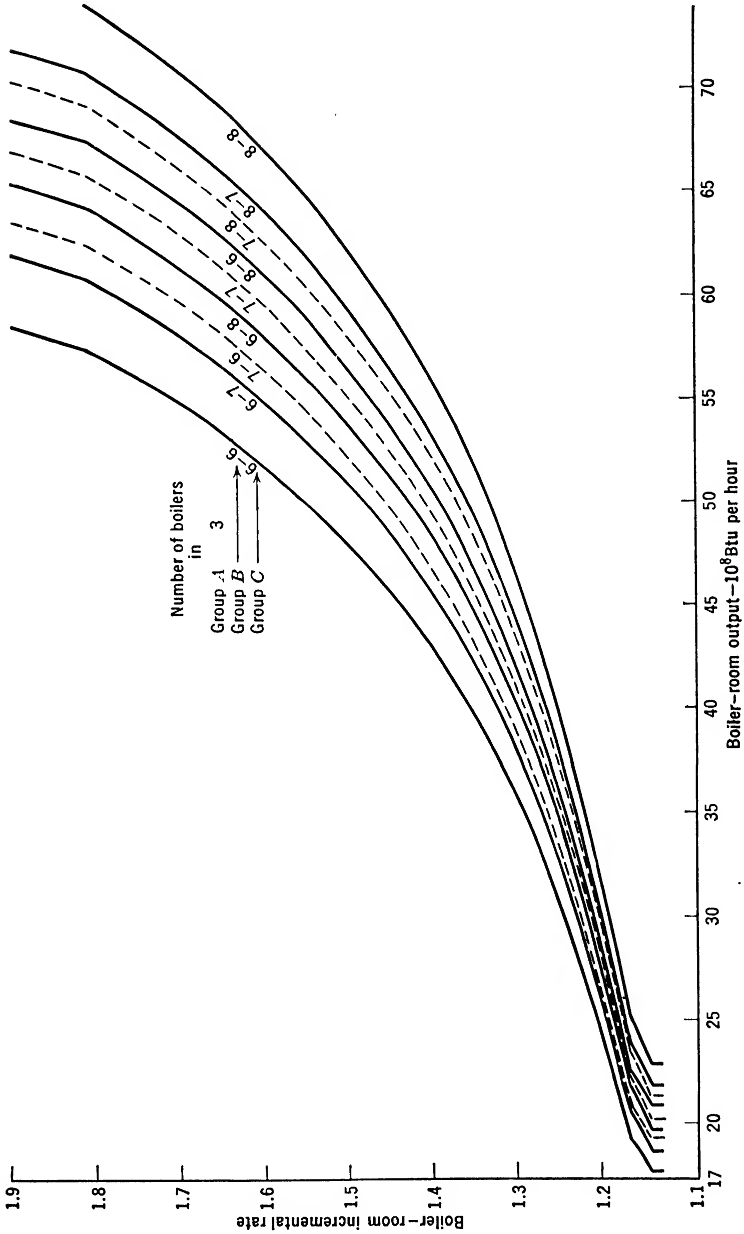


FIG. 27. Incremental rate curves for various combinations of boilers of Fig. 25.

same incremental rate values. Column 7 is the total output for all boilers. By plotting the incremental rates against the values of column 7, the overall boiler-room incremental rate curve is obtained for the particular combination of boilers as shown in Fig. 26.

In Fig. 27 is shown a series of boiler incremental rate curves for various combinations of boilers which are likely to be in service in the station. By having these curves available, it becomes a relatively simple matter to combine them with similar curves for the turbine room to derive the station incremental heat rate curve for any combination of turbine-generators and boilers in service. This phase of the subject will be treated in Chapter IV.

By establishing the curves of Figs. 25 and 27, the load division in the boiler room for any combination of boilers in service is possible. This is illustrated by the following example.

Example. Determine the load division in the boiler room for an output of 5,900 $\times$ 10^6 Btu per hour for the following combination of boilers:

3 boilers in Group A
7 boilers in Group B
8 boilers in Group C

From Fig. 27, at an output of 5,900 $\times$ 10^6 Btu per hour, the incremental rate is 1.50.

From Fig. 25, at an incremental rate of 1.50, the outputs should be

Group	Output—10^6 Btu per Hour	
	Per Boiler	Per Group
A	186	558
B	286	2,002
C	417.5	3,340
Total		5,900

Effect of Efficiency Curve Characteristic on Banking Procedure

Consideration of the load-division problems in the boiler room has so far been limited to the condition when all the boilers in service are operating under load. It is very probable that there will be periods during which it will be necessary, because of reduced loads on the station, to bank some of the boilers. During such periods the boiler-room operator is primarily interested in establishing a boiler banking sequence so that the highest level of overall boiler-room efficiency may be maintained. To attain this objective it is necessary to consider the nature of the individual boiler efficiency curves.

Analysis of boiler efficiency curves indicates that they may be classified with reference to the steaming rate at which the maximum efficiency occurs. In the first class, which will be referred to as type A, the maximum boiler efficiency occurs at the established minimum steaming rate.

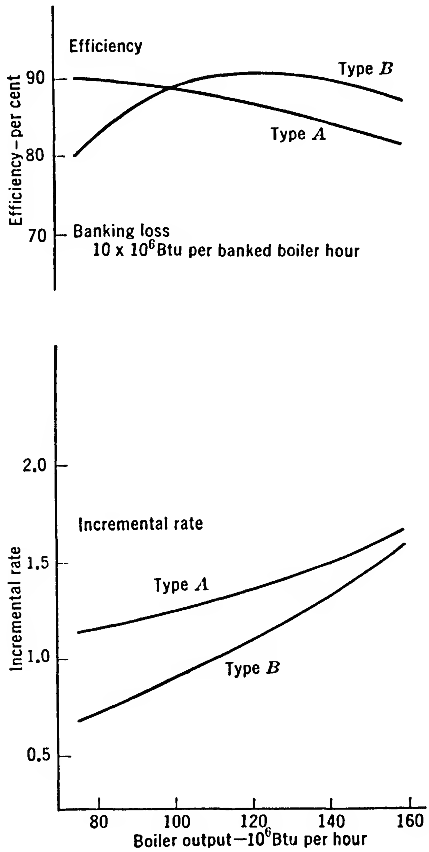


FIG. 28. Performance curves of type A and type B boilers.

In the second class, referred to as type B, the maximum boiler efficiency occurs at some steaming rate greater than the established minimum.

When all the boilers are identical and have the characteristics of type A, as many boilers should be kept steaming at all station loads as can be operated at or above the minimum rate. The total loads, therefore, at which boilers of this type should be banked are multiples of the minimum steaming rate of a single boiler.

The proper loads at which to bank type B boilers should be determined by the points of intersection of the overall efficiency curves for successive combinations of steaming and banked boilers.

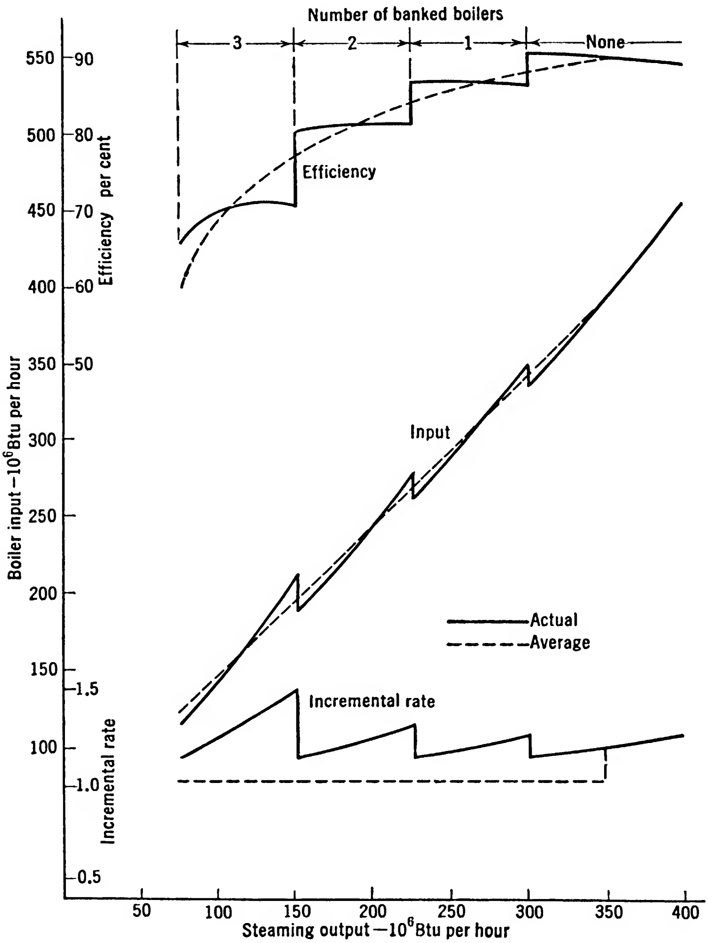


FIG. 29. Banking sequence for type A boilers.

Using the performance curves of Fig. 28, as representative of these types, the overall performance curves for the range of load requiring the banking of boilers are shown in Fig. 29 for the type A boilers, and in Fig. 30 for the type B boilers. The solid lines represent the overall performance when the banking sequence is in accordance with the procedure stated above. It will be noted that, for the type A boilers, both the input-output and efficiency curves are discontinuous at the points at

which boilers are banked, this being due to the change in banking loss. For the type B boilers, the input-output and efficiency curves are continuous, because, as a boiler is banked, the increase in the banking loss is

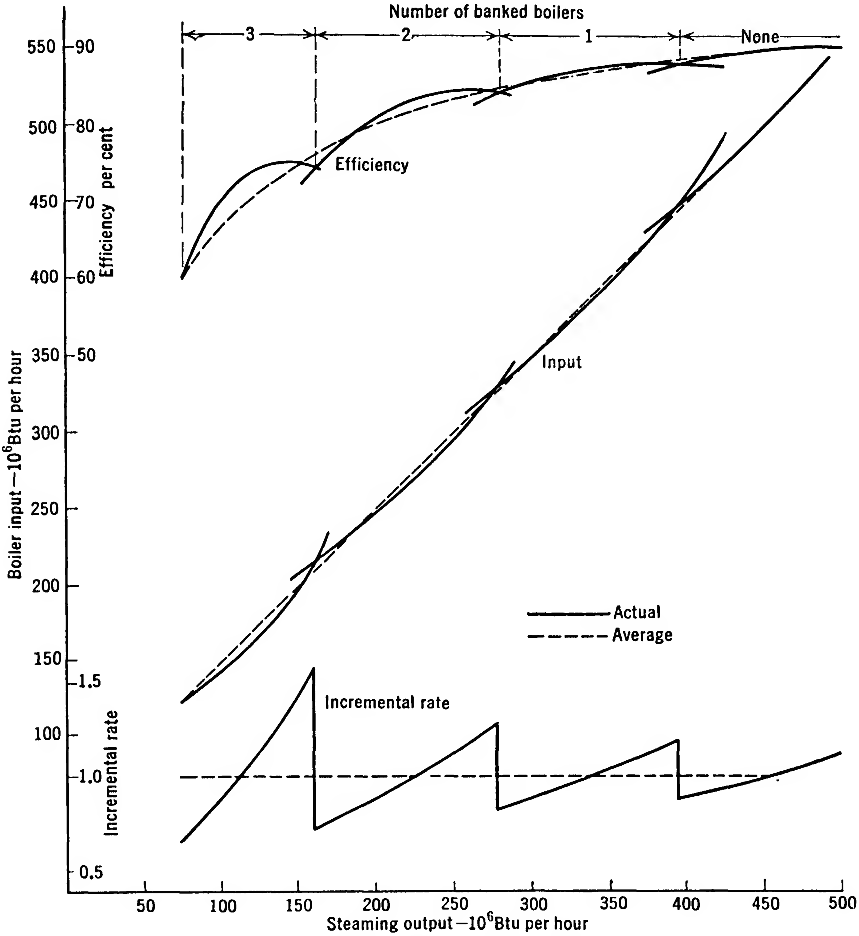
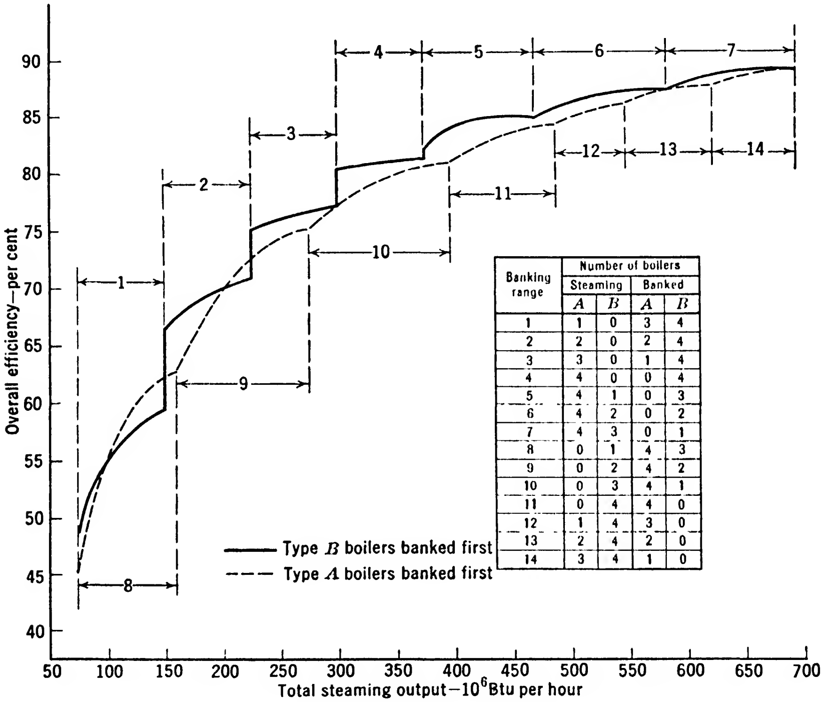


FIG. 30. Banking sequence for type B boilers.

offset by the decreased input to the steaming boilers resulting from their operation at a higher efficiency.

When groups of boilers operated in parallel differ in size and performance characteristics, it will be necessary to determine not only when to bank boilers but also which ones to bank to give the maximum efficiency. Often there is some operating reason for banking certain boilers in preference to others; if not, the various combinations should be computed to

determine which give the best results. The factors that determine which type of boiler to bank are the relative banking losses, the relative magnitudes and shapes of the efficiency curves, and the number of boilers of each type. It may be stated that, if the size, maximum efficiency, minimum steaming rate, and the banked loss of each type of boiler are substantially the same, it will usually be more efficient to bank the type B rather than the type A boilers.



Banking range	Number of boilers: Steaming A	Steaming B	Banked A	Banked B
1	1	0	3	4
2	2	0	2	4
3	3	0	1	4
4	4	0	0	4
5	4	1	0	3
6	4	2	0	2
7	4	3	0	1
8	0	1	4	3
9	0	2	4	2
10	0	3	4	1
11	0	4	4	0
12	1	4	3	0
13	2	4	2	0
14	3	4	1	0

FIG. 31. Banking sequence for a combination of type A and type B boilers.

In Fig. 31 are shown two series of efficiency curves obtained in the banked-boiler region for eight boilers, four of which are of type A and four of type B. The effect of banking all the type B boilers before any of the type A boilers is shown by the solid curve; the effect of banking all the type A boilers first is shown by the dotted curve. The difference in performance shows that, if the banking period is of considerable duration, the use of the correct sequence of banking is important.

Incremental Rate Curve for Banked Region

The proper sequence for banking boilers having been established, the next step is to derive the incremental rate curve for the range of load

which requires the banking of boilers. If, in a group of similar boilers, some are steaming and some are banked, then the *actual* incremental rate of the group is equal to the incremental rate of an individual steaming boiler; since the input to the banked boilers is a constant, it has no effect upon the actual incremental rate of the group. As the combined output of the group increases, a point is reached at which it is more economical to start steaming one of the banked boilers. At this point the incremental rate for the group decreases, and in many cases the input also decreases, especially when the boiler taken off bank is of type A. Adding another boiler allows a reduction in the individual outputs of the steaming boilers and hence a reduction in the incremental rate; if there is a decrease in total input it is because the reduction in input to the group of steaming boilers is greater than the increase in input to the boiler brought up from bank.

The decrease in the incremental rate values at the points of discontinuity will be reflected in the incremental rate curve for the station, so that, for the correct division of load between stations, the procedure outlined in Chapter I will be required. It frequently happens, however, that the range of load, in which the banked and steaming boiler combinations change, exists for only a relatively short period of time during which the load changes rapidly and the problem of load division is not of primary importance. For this transitory period the use of an average incremental rate value simplifies the calculations and in general is satisfactory.

The manner of establishing the average incremental rate value for the range of output requiring the banking of boilers depends upon the characteristics of the boiler efficiency curve. For the type A boilers the average should never be established from the actual incremental rate curve. This follows from the fact that an average faired through the actual incremental rate curve does not take into account the change in input corresponding to the point of discontinuity on the incremental rate curve. Hence the correct procedure requires that an average, generally a straight line, be faired through the input-output curve, resulting in an average incremental rate having a constant value.

For the type B boilers, as there are no points of discontinuity on the input-output curve, the average incremental rate may be established as the slope of the straight line faired through the input-output curve or a straight line faired through the actual incremental rate curve.

A comparison between the actual and average performance curves for the two types of boilers is shown in Figs. 29 and 30. When both types of boilers are involved the better practice would be to establish the average incremental rate as the slope of the straight line faired through the input-output curve.

Adjustment for Fuel Price Differentials

If the price of the fuel burned is the same for all boilers, operation of the boiler room to give the minimum heat input, or maximum overall efficiency, also makes the fuel cost for the station a minimum. Under such conditions it is sufficient to express the values of input and output in Btu per hour.

If the price of the fuels used by different groups of boilers is not the same, then the load division which gives a minimum heat input does not result in a minimum station fuel cost, and adjustment of the respective incremental rate curves is necessary. Two methods by which this can be done are:

1. The incremental rate curves for the individual boilers can be converted to incremental fuel cost curves directly, by multiplying the incremental rate values by the fuel prices. For example, if, at some given boiler output, the incremental rate is 1.2 and the fuel burned costs 20 cents per million Btu, then the incremental fuel cost at this output would be $1.2 \times 20 = 24$ cents per million Btu.

2. In practice it is simpler to adjust the incremental rate curves for differences in fuel prices than to convert the incremental rate curves. The adjustment is made by multiplying the incremental rate values by the ratio of the fuel price to some standard price. If the price of fuel used in one particular boiler is made the standard or base price, then the curves of all other boilers are adjusted to this base fuel price.

Consider three boilers which burn different grades of coal having the following prices:

Boiler	*A*	*B*	*C*
Cost of coal per short ton	\$3.60	\$4.25	\$4.75
Btu per pound of coal	13,000	14,000	14,200
Cost of coal per million Btu, cents	13.85	15.18	16.73

The price of the coal burned by boiler A will be selected as the base coal price. Therefore, the adjustment can be made by multiplying the incremental rate values of boiler B by $15.18 \div 13.85 = 1.096$ and those of boiler C by $16.73 \div 13.85 = 1.208$.

Calculation of Overall Boiler-Room Efficiency Curve

There are occasions when it is desirable to have available the overall efficiency curves for the boiler room for various combinations of boilers. They can be made by using the incremental rate curve for the particular combination of boilers and the overall efficiency for the group at some particular value of output.

It will first be assumed that all the boilers burn the same grade of fuel so that no price differential exists. In Fig. 32 are shown the performance curves for the individual boilers and for the group. The first step is the calculation of the overall boiler efficiency at some given output, say the

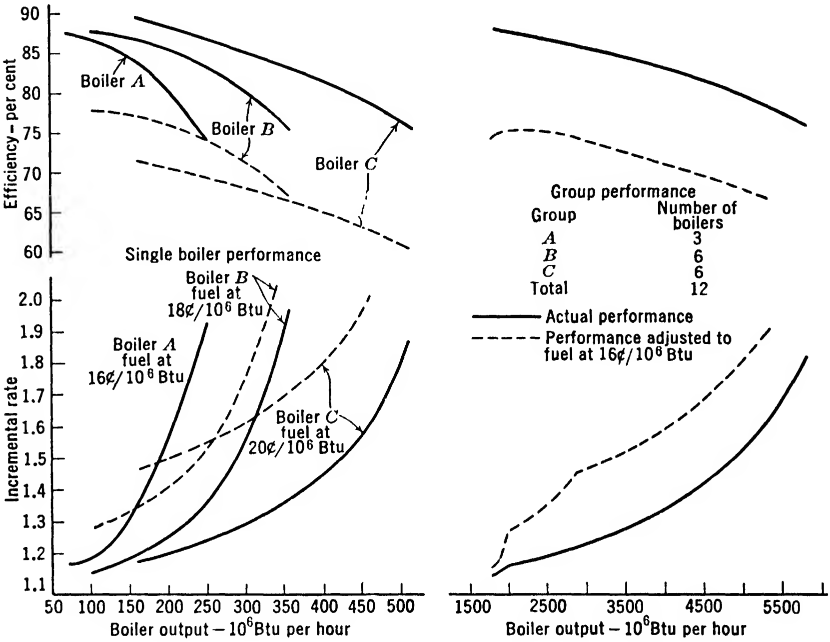


FIG. 32. Adjustment of boiler performance curves for fuel price differentials.

minimum steaming rate for all the boilers as a group, which is illustrated by the following tabulation:

Group	*A*	*B*	*C*	TOTAL
Number of boilers	3	6	6	15
Output per boiler, 10^6 Btu per hour	70	100	160	
Total output, 10^6 Btu per hour	210	600	960	1770
Efficiency, per cent	87.5	87.5	89.6	
Total input, 10^6 Btu per hour	240	683	1071	1994

The overall efficiency when all boilers are at the minimum steaming rate becomes $(1{,}770 \div 1{,}994) \times 100 = 88.8$ per cent.

Table X illustrates the next step in the procedure. In column 1 are tabulated the total outputs, the values being selected so that (except for the first) a constant difference is obtained, as shown in column 2. The average incremental rates of column 3 are obtained from the overall

TABLE X

CALCULATION OF EFFICIENCY CURVE FOR A GROUP OF BOILERS

Total Output 10^6 Btu per Hour	Incremental Output (ΔO) 10^6 Btu per Hour	Average Incremental Rate $\left(\frac{\Delta I}{\Delta O}\right)$	Incremental Input (ΔI) 10^6 Btu per Hour	Total Input 10^6 Btu per Hour	Overall Efficiency Per Cent
(1)	(2)	(3)	(4)	(5)	(6)
(1,770)				(1,994)	(88 8)
	230	1.158	266		
2,000				2,260	88.5
	200	1.177	235		
2,200				2,495	88.2
	200	1.190	238		
2,400				2,733	87.8
	200	1.205	241		
2,600				2,974	87.4
	200	1.220	244		
2.800				3,218	87.0
	200	1.235	247		
3,000				3,465	86.6
	200	1.252	250		
3,200				3,715	86.1
	200	1.270	254		
3,400				3,969	85.7
	200	1.292	258		
3,600				4,227	85.2
	200	1.315	263		
3,800				4,490	84.6
	200	1.340	268		
4,000				4,758	84.1
	200	1.370	274		
4,200				5,032	83.5
	200	1.403	281		
4,400				5,313	82.8
	200	1.440	288		
4,600				5,601	82.1
	200	1.480	296		
4,800				5,897	81.4
	200	1.527	305		
5,000				6,202	80.6
	200	1.583	317		
5,200				6,519	79.8
	200	1.645	329		
5,400				6,848	78.9
	200	1.715	343		
5,600				7,191	77.9
	200	1.795	359		
5,800				7,550	76.8

incremental rate curve of Fig. 32. For example, the incremental rate of 1.19 is the average value for the increment of output between 2,200 and 2,400 and is read directly from the curve at an output of 2,300, which is the average output for the interval. This is permissible when the intervals of output are made small enough so that the curve may be considered to be a straight-line function between the successive outputs. Column 4 is obtained as the product of the corresponding values of columns 2 and 3. The total inputs are shown in column 5. Commencing with the known input, shown in parentheses, the values of column 4 are cumulatively added for increasing outputs. The overall efficiencies are then obtained by dividing the values of column 1 by those of column 5 and plotted against the values of total output as shown in Fig. 32 by the solid curve.

It has been shown that a boiler incremental rate curve can be adjusted to a base fuel price by multiplying the actual incremental rate values by the ratio of the actual price to the base price. The efficiency curve corresponding to the incremental rate curve adjusted to the base fuel price can be obtained by dividing the actual efficiency values by the same ratio. The adjusted efficiency does not define the actual thermal performance of the boiler, but its use in conjunction with the base fuel price will permit the calculation of the actual fuel costs at any output. The following illustration demonstrates this:

Boiler output	100×10^6 Btu per hour
Actual efficiency	80 per cent
Actual fuel price	\$0.20 per 10^6 Btu
Base fuel price	\$0.16 per 10^6 Btu
Adjustment factor (20 ÷ 16)	1.25
Efficiency adjusted to base fuel price (80 ÷ 1.25)	64 per cent

Using actual values, the fuel cost at the given output becomes

$$\frac{100}{0.80} \times \$0.20 = \$25 \text{ per hour}$$

Using adjusted values, the same fuel cost is obtained from

$$\frac{100}{0.64} \times \$0.16 = \$25 \text{ per hour}$$

To obtain the actual fuel costs for a boiler plant in which fuel at different prices is used it is expedient to establish the overall boiler-room efficiency adjusted to a base fuel price. This can be done by first adjusting the incremental and efficiency curves of the individual boilers to the base fuel price. The incremental rate curve for any combination of

TABLE XI

Effect of Fuel Price Adjustments on Boiler-Room Performance

	Without Fuel Price Adjustment				With Fuel Price Adjustment			
Group	*A*	*B*	*C*	Total	*A*	*B*	*C*	Total
Number of boilers	3	6	6	15	3	6	6	15
Output, 10^6 Btu per hour								
Per boiler	149	227	311		197	257	257	
Per group	447	1,362	1,866	3,675	591	1,542	1,542	3,675
Boiler efficiency (actual) per cent	84.8	84.7	85.0	84.87	80.6	83.2	86.7	84.19
Input per group	527.1	1,608.0	2,195.3	4,330.4	733.3	1,853.3	1,778.5	4,365.1
Price of fuel burned, cents per 10^6 Btu	16	18	20	18.77*	16	18	20	18.48*
Fuel cost per hour	$84.34	$289.44	$439.06	$812.84	$117.33	$333.59	$355.70	$806.62

* Average price of fuel burned.

boilers can then be derived from the adjusted incremental rate curves of the individual boilers in the manner illustrated in Table VIII or Table IX. The adjusted efficiency curve for the particular combination of boilers can be established in the manner illustrated in Table X by using the adjusted values instead of actual values.

Figure 32 illustrates the actual and adjusted performance curves for three groups of boilers based on the following fuel prices:

Group	*A*	*B*	*C*
Price of fuel burned, cents per 10^6 Btu	16.0	18.0	20.0
Correction factor	1.000	1.125	1.250

A comparison of the operating results with and without adjustment for fuel price differentials is illustrated in Table XI for the group of boilers represented by the curves of Fig. 32, for an output of 3,675 million Btu per hour. Referring to Table XI it will be noted that loading the boilers on the basis of their respective incremental rates adjusted for fuel price differentials reduces the overall thermal efficiency of the boiler room from 84.87 to 84.19 per cent. There is, however, a reduction in the cost of fuel burned to deliver the given boiler-room load. Since the fuel cost and not the thermal efficiency is the true measure of efficient operation, it follows that the adjusted incremental rate curves constitute the proper basis for loading the boilers.

The hourly fuel costs for any given boiler-room output can be derived from the adjusted overall efficiency curve without resorting to the calculations indicated in Table XI. For example, at the output of 3,675 million Btu per hour, the adjusted efficiency, read from the curve of Fig. 32, is 72.9 per cent. This is the efficiency adjusted to a fuel price of 16 cents per million Btu. Hence the hourly fuel cost becomes

$$\frac{3675}{0.729} \times \$0.16 = \$806.59 \text{ per hour}$$

This compares with the fuel cost of $806.62 per hour indicated in Table XI, the difference being well within the accuracy with which values can be read from the curves of Fig. 32.

CHAPTER III

LOAD DIVISION IN THE TURBINE ROOM

Introduction

The problem of load division in the turbine room is more complex than that in the boiler room. There is ordinarily a much wider choice as to the number of units to be operated and the sequence of adding units as the load increases. Whereas the number of boilers in service is commonly determined by the number out for repairs or overhauling, turbines are on the line part of the day or all day as the station load demands. In the boiler room the problem is how much load each type of boiler shall carry. In the turbine room it is how many units shall be put in service; then which units shall be used; and, finally, how the load shall be divided among them.

The variation in types of turbine units installed in a given station is often greater than the variation of the boilers in the same station. Each unit may be different in size and type. There may be impulse and reaction turbines; straight condensing and extraction at one or several stages; constant and variable feedwater temperature; single-cylinder, tandem-compound, and cross-compound.

The heat balance may be further complicated by the kind of auxiliaries. If they are electrically driven, there may be three sources of supply: house turbine-generators; house generators on the same shafts as the main turbine-generators; or house transformers. A station may use all three sources of supply.

If the auxiliaries are steam driven, part or all of the exhaust steam may be utilized to heat the feedwater. This is done in many older stations where the main units are operated straight condensing. In some modern plants exhaust and extraction steam are used for feed heating. Since the exhaust steam is at approximately atmospheric pressure, it will usually enter the cycle between steam extracted at lower and higher pressures; this makes the turbine-generator performance dependent upon the amount of exhaust steam used for feed heating.

The Heat Rate Curve

The turbine-generator heat rate curve defines the performance characteristic of the unit and it is the basic curve from which the incremental

rates are derived for load-division purposes. The heat rate of a unit serves as a means of indicating its thermal efficiency, since

$$\text{Thermal efficiency in per cent} = \frac{3412.75}{\text{Heat rate}} \times 100$$

If the problem of load division in the turbine room is to be solved satisfactorily, it is essential that the heat rate curves for the respective units be accurately established for the conditions under which each unit actually operates. Data determined by tests under actual operating conditions are the most reliable and should be obtained whenever possible. Guarantee data should not be used unless unavoidable since frequently the actual performance is considerably different.

Tests on extraction-type units should be made under normal extraction conditions. If test facilities do not permit this, an approximation of feed-heating performance can be calculated from a careful non-extraction test. A practical method for computing extraction cycles is outlined in the paper "Economy Characteristics of Stage Feedwater Heating by Extraction" by Messrs. E. H. Brown and M. K. Drewry, published in Vol. 45 of the *A.S.M.E. Transactions* for 1923. Application of the procedure discussed in this paper has been found to be satisfactory, and, if readings in the field of the various pressures and temperatures can be obtained while the unit is operating under its normal feed-heating cycle, a computed performance within 2 per cent of test results is possible.

Turbine Exhaust Pressure

The efficiency of a turbine-generator, as reflected by its heat rate, is influenced by the pressure and temperature at the throttle and by the pressure at the turbine exhaust. In general, the steam pressure at the turbine throttle is maintained so nearly constant that it may be neglected as a variable influence on the performance of the turbine. The temperature of the steam changes, however, depending on the number of boilers in service and the loads at which they are operating. The variation of steam temperature may be disregarded as a factor by correcting the performance of the turbine-generator to the standard temperature to which the boiler efficiency curves are adjusted, as described in Chapter II.

This leaves for consideration the influence of the turbine exhaust pressure on the performance of the unit. The pressure which can be maintained at the turbine exhaust is influenced by the following:

1. The load on the generator.
2. The quantity of steam exhausted to the condenser.
3. The cleanliness of the condenser tubes.
4. The amount of air leakage into the condenser.

5. The quantity of circulating water.
6. The inlet temperature of the circulating water.

These can be directly or indirectly controlled, with the exception of the inlet circulating water temperature, which generally varies with the

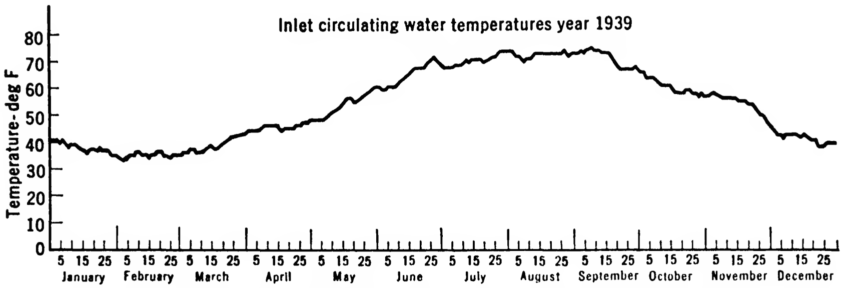


FIG. 33. Seasonal variation of inlet circulating water temperature.

seasons of the year, as shown in Fig. 33 for a station in the middle Atlantic states. The relation between the turbine-generator heat rates and the inlet circulating water temperatures should be established so that the effect of the seasonal variation of the water temperature on the performance of the unit may be accounted for. The procedure for this is illustrated by Figs. 34 to 37 inclusive.

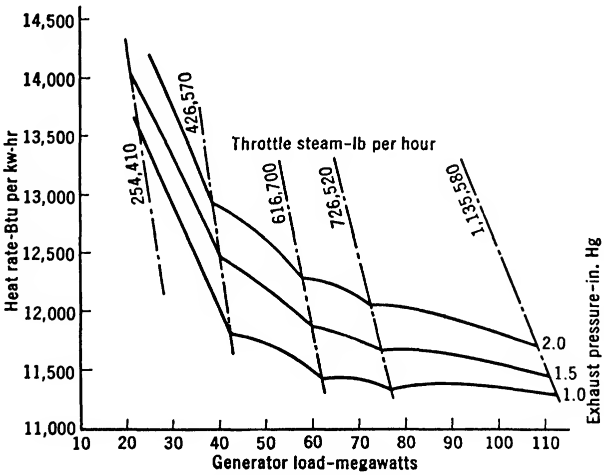


FIG. 34. Turbine-generator heat rate curves at constant exhaust pressures.

Heat rate curves for each turbine-generator unit should be established for several values of exhaust pressure. A typical set of such curves is shown in Fig. 34 for a unit having four steam admission valves. The load which can be generated with any group of admission valves fully

open is represented by the lines of constant steam flow. It is to be noted that, with constant steam flow, a change in exhaust pressure changes not

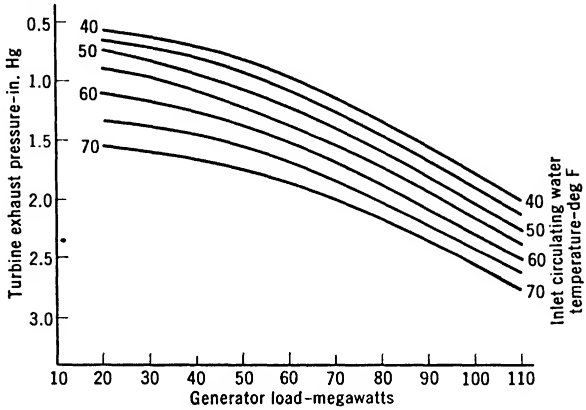


FIG. 35. Turbine exhaust pressures at constant inlet circulating water temperatures.

only the heat rate of the unit but also the load that can be generated. This is an important point to keep in mind, for, as will subsequently be shown, the calculation of turbine-generator incremental heat rates can be greatly simplified by considering the performance of the unit at only those loads at which the steam admission valves are fully open.

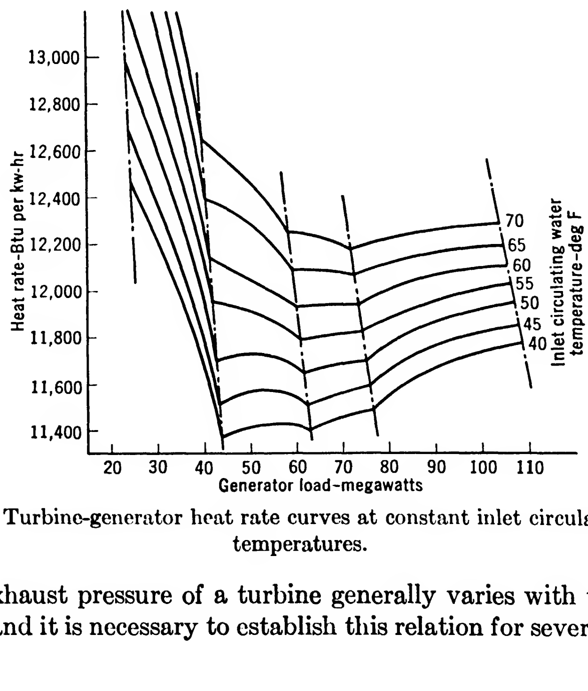


FIG. 36. Turbine-generator heat rate curves at constant inlet circulating water temperatures.

The exhaust pressure of a turbine generally varies with the load on the unit, and it is necessary to establish this relation for several values of

inlet circulating water temperature, as illustrated in Fig. 35 for the unit in question. Data for these curves are generally obtained from the operating log sheets; if no records are maintained, readings in the field will be required. In plotting the exhaust pressures against the generator loads for the given circulating water inlet temperature, it will be generally found that the plotted points will lie within a band representing upper and lower limits. This may be due to errors in reading the instruments, variation in condenser-tube cleanliness, air leakage, and quantity of

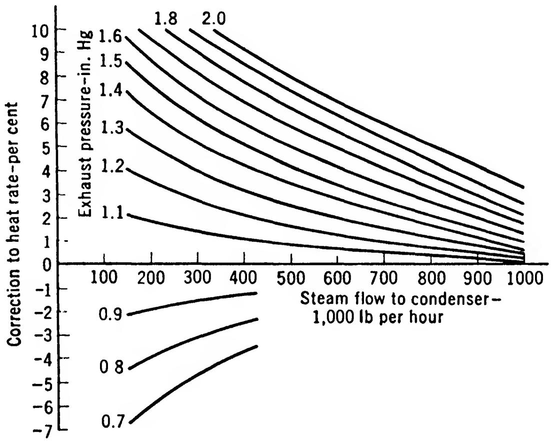


FIG. 37. Corrections to turbine-generator heat rate for variation of turbine exhaust pressure.

circulating water. To establish the curve as representative of average conditions it should be drawn so that it will lie approximately midway between the upper and lower limits of the band.

From the curves of Figs. 34 and 35 it is a simple matter to establish the curves of Fig. 36, which are in a form suitable for establishing the turbine-generator incremental rates as functions of the inlet temperature of the condenser circulating water.

Although data for applying corrections to the turbine-generator heat rates for variation of exhaust pressure are usually supplied by the manufacturer, they should preferably be verified by tests. One of several acceptable ways of showing them is illustrated by the curves of Fig. 37.

Adjustment for Steam-Driven Auxiliaries

When sufficient and reasonably accurate data on individual auxiliary requirements are available, no involved procedure is required to adjust the individual turbine-generator input-output curves to include the auxiliary heat inputs. For each turbine, the input for any given load is

increased by the corresponding amount of the auxiliary heat consumption to give the total requirement from the boilers, as illustrated in Fig. 38. This can be done, however, only if the heat consumption of the auxiliaries can be segregated for the individual boilers and turbines, and

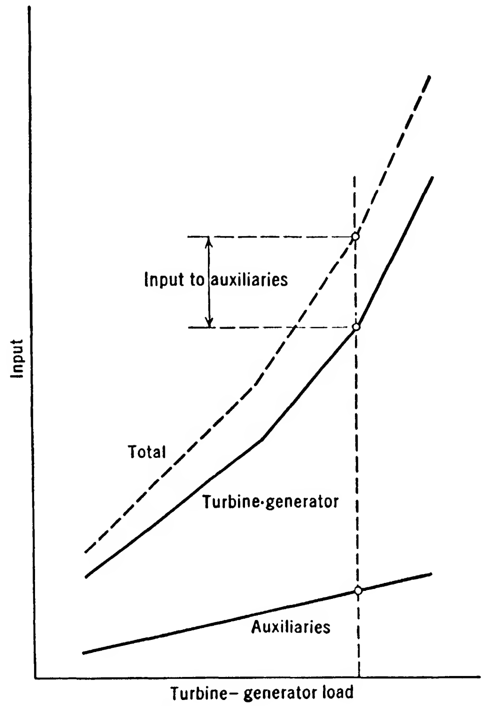


FIG. 38. Adjustment of turbine-generator input-output curve for steam auxiliary consumption.

if the exhaust steam is utilized after the turbine cycle is complete. A typical heat balance cycle which satisfies these conditions is illustrated in Fig. 39.

In Fig. 40, however, is shown a heat-balance diagram in which the exhaust steam from a station header enters the turbine cycle before a heater in which extracted steam is used. For a cycle of this kind, the heat consumption of the turbine from which extraction is made is affected by the amount and quality of the exhaust steam, since these control the temperature of the feedwater entering the extraction heater and thereby determine the amount of extracted steam. Hence, resort would have to be made to cut-and-try methods if it were desired to compensate for the variations in heat demand of the auxiliaries of various units. The

performance of any turbine would be affected by the number and loading of other turbines, and by the number and loading of the boilers in service.

An approximate solution could be made by establishing for each turbine two curves, both plotted against the turbine-generator load. One

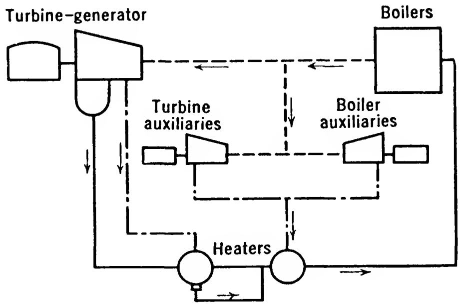


FIG. 39. Heat balance which permits segregation of boiler and turbine auxiliary steam consumption.

would show the heat available for feedwater heating in the exhaust steam of the turbine-generator auxiliaries. The other, showing the heat available in the exhaust of the boiler auxiliaries, would be an approximate one, empirically determined by dividing the heat consumption of the boiler auxiliaries among the individual turbine-generators, so that, for the normal sequence of operation, the sum of the quantities so assigned would equal the total heat consumption of the boiler auxiliaries through

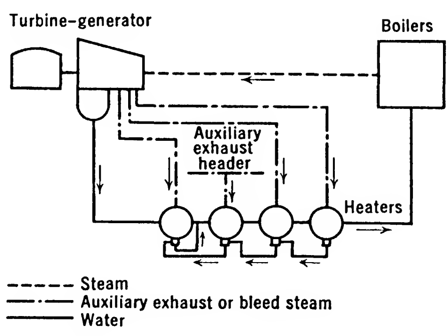


FIG. 40. Heat balance which does not permit segregation of boiler and turbine auxiliary steam consumption.

all ranges of station load. The two curves would be used to determine the input-output curve for each turbine. Once these were derived, the equivalent input-output curves of boilers and turbines would be computed in the manner already described.

Adjustment for Electrically Driven Auxiliaries

Adjustment of the turbine-generator heat rate curve for the power consumption of electrically driven auxiliaries depends upon the source of supply to the auxiliaries. One source of supply is through house transformers. The effect of supplying station auxiliaries from this source is to reduce the energy available for supplying the system load. For each turbine, therefore, the adjustment is made by subtracting, from the generator output, the corresponding amount of auxiliary power consumption. Dividing the turbine input, which remains constant, by the reduced or net output of the generator gives the net heat rate for the unit. The turbine net incremental heat rates should be plotted against the generator load for use in dividing load among the turbines, the procedure being similar to that described for adjusting the boiler efficiency curve for auxiliary power consumption.

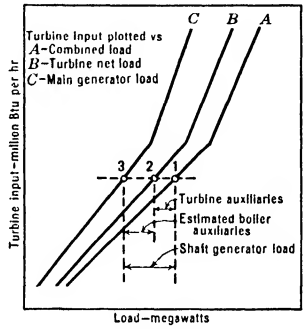


FIG. 41. Adjustment of turbine-generator input-output curve for auxiliary power consumption and load on shaft generator.

When station auxiliary power is obtained from house generators on the shafts of the main turbines, it may be difficult to establish an exact relation between the outputs of the house and main generators. Since the output of the house generator is small compared to that of the main generator, the error in assuming its efficiency to be equal to that of the main generator is negligible. Thus it may be assumed that the heat input to the turbine is a function of the total load on the two generators and is independent of the division of load between them.

The net turbine-generator load, which is the combined load less the electric power consumption of the turbine auxiliaries, should be used in finding the turbine incremental rate. Since the actual assignment of load to a unit is usually made in terms of load on the main generator, the division of load between the main and shaft generators must be estimated. This can be done by assuming average conditions in the boiler room and allocating to each unit a share of the boiler-room electric auxiliary power, which, added to the turbine-generator auxiliary power, gives the shaft generator load. Referring to Fig. 41, curves A, B, and C represent the input to the turbine plotted, respectively, against the combined load, the turbine-generator net load, and the main generator load. For purposes of load division in the turbine room, the incremental rates of curve B should be plotted against the corresponding load values of curve

C. Thus the incremental rate at point 2 will be plotted against the load at point 3. The resulting curve would then show the turbine-generator net incremental rate plotted against the main generator load, and it should be used in conjunction with the incremental rate curves of other turbine-generators to determine the turbine loading.

The heat input to house turbine-generators which supply electrically driven auxiliaries should be treated in the same manner as steam-driven auxiliaries. The input to the house unit should be divided among the boilers and turbines in proportion to the power consumption of their respective auxiliaries, and the adjustment should be made as described for steam-driven auxiliaries.

Adjustment for Feedwater Temperature

Variation in the temperature of the feedwater supplied to the boilers is generally due to a turbine heat-balance cycle which employs extraction steam for feedwater heating. A variable feedwater temperature may affect the efficiency of the turbine-generator only, or the efficiency of both the boiler and turbine-generator, depending upon whether or not the boilers are equipped with economizers.

In a station having no economizers on the boilers, the change in superheat with change in feedwater temperature is the only consideration. Some knowledge of the relationship existing between the feedwater temperature and superheat for the particular boilers is necessary. If the boiler performance curves have been adjusted to a standard steam temperature T_s and a standard feedwater temperature t_s in the manner described in the preceding chapter, and the turbine-generator performance curves have been established for the same standard steam temperature T_s and for the actual feedwater temperature t_a, then the turbine-generator performance need be corrected only for the effect of its feedwater temperature characteristic on the superheat of the steam generated in the boilers, and hence on its own economy. The method of making this adjustment is illustrated in Table XII, and the effect of the adjustment is shown in Fig. 42 for a 50,000-kw unit with two stages of feed heating. Referring to the figure, the solid curve represents the heat rate for the unit when the throttle steam temperature is at the standard value T_s and the temperature of the feedwater supplied by the unit is at the actual value t_a. The dotted curve is the heat rate for the unit adjusted to the temperature of the steam T'_a which the boilers would supply if receiving feedwater at the actual temperature t_a.

The adjustment discussed above is based on the assumption that a relatively small change in throttle steam temperature does not materially change the temperature of the feedwater supplied by the unit.

TABLE XII

ADJUSTMENT OF TURBINE-GENERATOR HEAT RATE FOR VARIABLE FEEDWATER TEMPERATURE

Load Mw	t_a	$t_s - t_a$	$T_s - T'_a$	T'_a	$1 + k(T_s - T'_a)$	H'_a	q_a	$H'_a - q_a$	$H_s - q_a$	HR'_a/HR_s	HR_s	HR'_a
12	251	+29	− 7.3	607.3	0.9927	1,318.7	219.5	1,099.2	1,095.2	0.9963	15,000	14,945
18	263	+17	− 4.3	604.3	0.9957	1,317.0	231.7	1,085.3	1,083.0	0.9978	13,923	13,892
24	275	+ 5	− 1.3	601.3	0.9987	1,315.4	243.9	1,071.5	1,070.8	0.9994	13,385	13,377
30	286	− 6	+ 1.5	598.5	1.0015	1,313.9	255.2	1,058.7	1,059.5	1.0007	13,063	13,072
36	298	−18	+ 4.5	595.5	1.0045	1,312.2	267.5	1,044.7	1,047.2	1.0021	12,847	12,874
40	306	−26	+ 6.5	593.5	1.0065	1,311.1	275.8	1,035.3	1,038.9	1.0030	12,949	12,988
45	316	−36	+ 9.0	591.0	1.0090	1,309.8	286.1	1,023.7	1,028.6	1.0042	13,051	13,106
47	320	−40	+10.0	590.0	1.0100	1,309.2	290.3	1,018.9	1,024.4	1.0046	13,157	13,218
50	326	−46	+11.5	588.5	1.0115	1,308.4	296.5	1,011.9	1,018.2	1.0052	13,300	13,369

$t_s = 280°$ F

$T_s = 600°$ F

$H_s = 1314.7$ Btu per lb

$k = 1$ per cent for 10° F

Pressure = 300 psi abs

$T_s - T'_a = -0.25(t_s - t_a)$

$$\frac{HR'_a}{HR_s} = \frac{[1 + k(T_s - T'_a)](H'_a - q_a)}{(H_s - q_a)}$$

t_s and t_a are, respectively, the standard and actual feedwater temperatures.
HR_s = turbine-generator heat rate corresponding to T_s and t_a.
HR'_a = turbine-generator heat rate corresponding to T'_a and t_a.
T'_a = steam temperature obtained when boiler feedwater temperature is changed from t_s to t_a.

It is to be observed from the ratios of the adjusted to unadjusted heat rates, shown in Table XII as HR'_a/HR_s, that the maximum change in the heat rate of the unit is approximately ½ per cent, which for practical purposes may be small enough to justify omitting the correction as an unwarranted refinement.

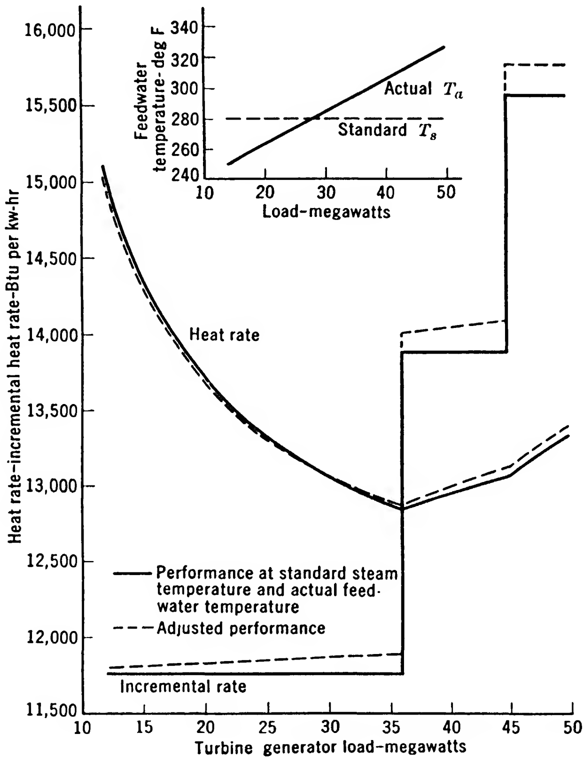


FIG. 42. Adjustment of turbine-generator performance curves for variation of final feedwater temperature.

When the boilers are equipped with economizers, a variable feedwater temperature will affect the efficiency of the boiler unit. The amount of heat recovered in the economizer changes considerably with change in feedwater temperature, so that the boiler efficiency is dependent upon this temperature. The effect of change in feedwater temperature on the boiler superheat, however, is less when there is an economizer than when the feedwater goes directly into the boiler drum. The change in superheat and change in efficiency of the boiler unit per degree Fahrenheit change in feedwater temperature into the economizer should be determined, preferably by test. When these relations are known, the boiler

efficiency curve can be established for the standard steam and feedwater temperature, in the manner previously described.

The turbine-generator heat rate curve is then adjusted in three steps:

1. The actual turbine-generator heat rate curve is corrected to the standard steam temperature and actual feedwater temperature.

2. The curve from step 1 is corrected for the effect, on the *turbine-generator* efficiency or heat rate, of the variation of the boiler superheat resulting from the difference of the actual turbine-feedwater temperature from the standard.

3. The curve from step 2 is adjusted for the effect, on the *boiler efficiency*, of the variation of the actual turbine feedwater temperature from the standard.

Heat Rate Curve Characteristics

The performance curves for turbine-generators usually fall within one of the three types illustrated in Figs. 43, 44, and 45. In Fig. 43, the

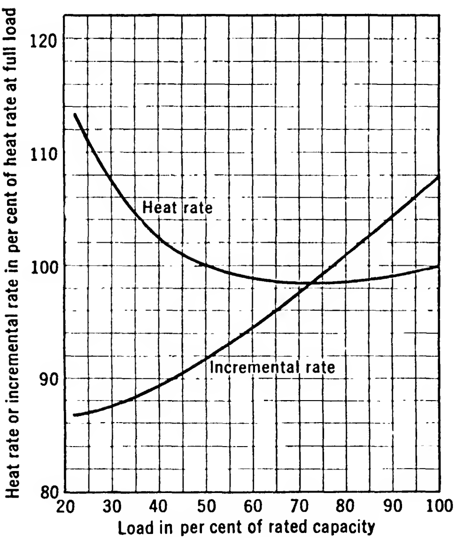


Fig. 43. Turbine-generator performance curves similar to Type I (Fig. 7).

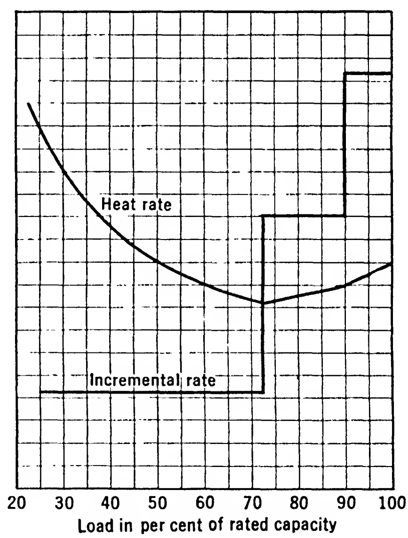


Fig. 44. Turbine-generator performance curves similar to Type III (Fig. 7).

heat rate and incremental rate curves are smooth and continuous, so that the input-output curve is also a smooth continuous curve.

The curves of Fig. 44 are typical for multivalved units designed for admission of steam to lower stages by by-passing the first few rows of blading. The heat rate curve consists of sections of smooth continuous curves, the number of sections being equal to the number of steam

admission valves, or groups of valves. The corresponding input-output curve consists of a series of intersecting straight lines, so that, between points of intersection, the incremental heat rates have constant values. This results in a discontinuous incremental rate curve with steplike increases at those loads at which one or more steam admission valves are fully open.[1]

The solid curves of Fig. 45 are also typical for a multivalved unit. The heat rate curve is continuous with breaks at the valve loads. Over

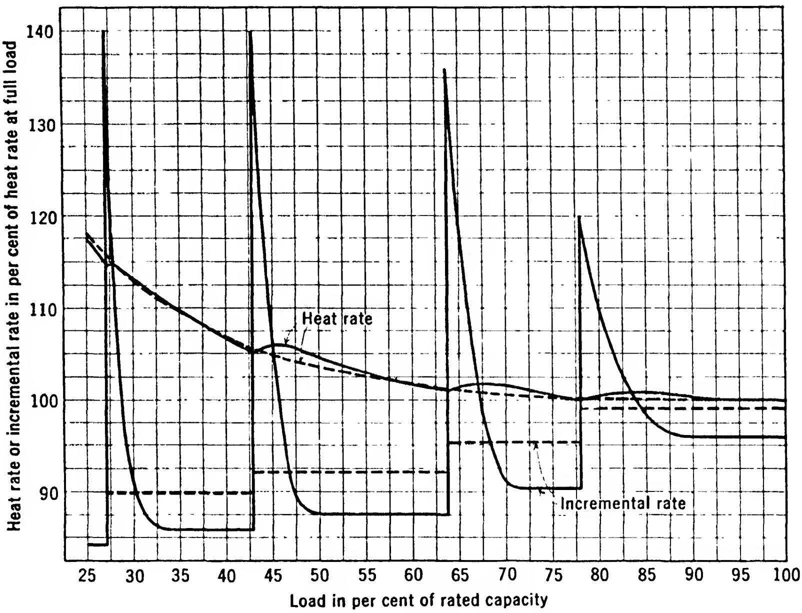


FIG. 45. Turbine-generator performance curves having peak incremental rate values

a short load range following each valve point, the heat rate curve has a slight hump, which causes the incremental rate to increase suddenly and then gradually decrease to a constant value until the next valve point is reached. Because of the sharp peaks in the incremental rate curves, it is not practicable to use the curve in this form for load division, and in the interest of convenience the use of average incremental rates may be resorted to without introducing more than a negligible error. The average incremental rates between the valve points are obtained by dividing

[1] Ordinarily, there is a slight range of overlap, so that one valve is not quite completely open before the succeeding valve begins to pass steam. Here, and throughout the text, the term "fully open valve" or "full valve opening" refers to the point at which the next valve begins to open.

the differences in successive inputs corresponding to the valve-opening loads by the corresponding differences in output. This is shown by the dotted curve of Fig. 45, and it is to be noted that the average curve assumes the form shown in Fig. 44.

There are two reasons justifying the use of average incremental rates for this type of unit. The first involves consideration of the accuracy

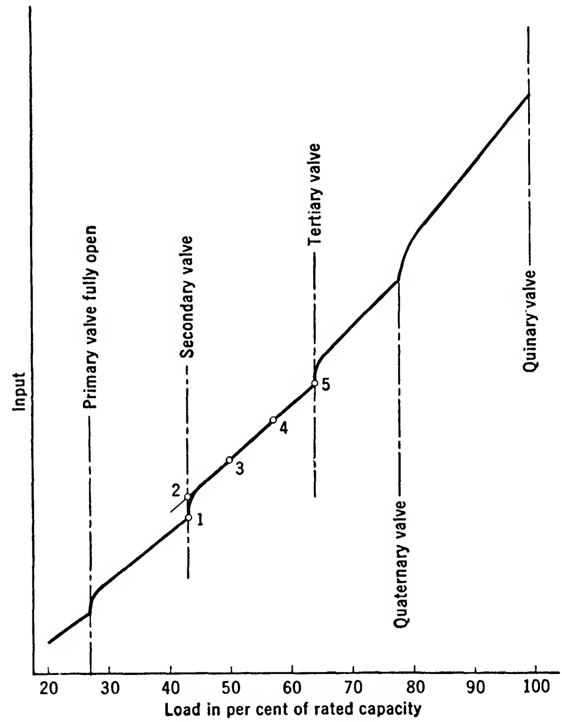


FIG. 46. Input-output curve corresponding to curves of Fig. 45.

with which the input-output curve can be established. Figure 46 shows schematically the input-output curve corresponding to the one of Fig. 45. The curve is usually drawn through a series of points obtained by test. For example, considering the range of load between 43 and 64, which is the load supplied by the tertiary valve, tests would be conducted at the valve loads of 43 and 64, and perhaps at two or more additional loads which are intermediary values, say at 50 and 57. There would, therefore, be four points through which the input-output curves should be drawn. Points 3, 4, and 5 would establish the input-output curve as a straight line which, if extended, would introduce a discontinuity represented by the distance between points 1 and 2. Since the curve is known

to be continuous, it becomes necessary to depart from a straight-line function and introduce a curvature between points 1 and 3 which causes the sharp peaks in the incremental rate curve.

The exact nature of the curvature is not capable of determination by ordinary methods of testing principally because it is very difficult to maintain valve openings constant in this range. Furthermore, the changes in heat input for small changes in valve setting may be within the magnitude of the total accuracy.

Assuming that the incremental heat rates corresponding to the curved portions of the input-output curve are capable of exact determination, there still remains the question of practicability. Suppose that it is desired to use the actual incremental rates for load division; then it will be found that for a given combined load there will be several combinations of loads which correspond to the same incremental rate value, and considerable computation may be required to determine the division of load for maximum combined efficiency.

To illustrate, consider the problem of load division as applied to two similar units with incremental and heat rate curves represented by the solid curves of Fig. 45. Suppose that the total load of 110 is to be divided between the two units; then the incremental rates for each unit will be equal for the following combination of loads: (27, 83), (28.25, 81.75), (43, 67), (44.5, 65.5), (46, 64), and for all combinations from (49, 61) to (55, 55).

Yet the minimum combined heat rate will be obtained for only one pair of loads, which will occur in this case when one unit is at a load of 46 and the other at 64. Figure 47 shows the effect of load division on the combined heat rate for both units for several loads. Since the two units are identical, each curve of Fig. 47 is symmetrical about the load corresponding to half of the combined load. The circles show, for each total load, the combination which gives the minimum combined heat rate.

Analysis of the curves of Fig. 47 indicates that the best combination results when one of the units is operating at the valve load which is most nearly one-half of the total load. It is also important to note that, for units having performance curves of the type shown in Fig. 45, the overall heat rate *is not a minimum when the load is equally divided among the identical units.* In fact, for a total load of 170, dividing the load equally gives the highest input of any possible combination.

There is another approach to the problem of loading machines of this type. To illustrate, consider two turbine-generator units having the performance characteristics shown in Fig. 48. These are similar but not identical. The incremental rates for these units have been plotted to show average values for incremental loads of 1 megawatt, which simpli-

fies the calculations without affecting the principle involved. An additional set of curves, shown in Fig. 49, is required. These curves are plotted for each valve to show the incremental rate at which an increment of load can be generated by steam admitted through the particular valve. The increment of load is measured from the load at which the preceding valve is fully open as the datum. For example, the secondary valve of each machine is fully open at a load of 90 megawatts. An increment of 5 megawatts supplied by the tertiary valve of one machine would

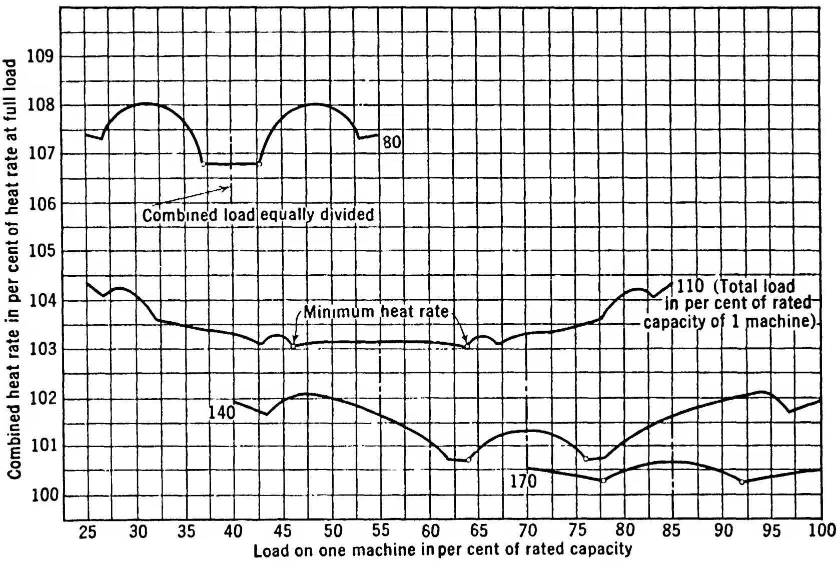


FIG. 47. Effect of load division on combined efficiency of two identical turbine-generators having performance curves of Fig. 45.

correspond to a load on that machine of 90 + 5 or 95 megawatts. Curves for the primary valves are not shown since the corresponding incremental rate values are constant.

The curves of Fig. 49 permit determination of the proper loading sequence by inspection. Assuming a minimum operating load of 40 megawatts for each machine, the loading sequence for the machines in question would be first to load unit *B* to its primary valve load of 60 megawatts and then unit *A* to its primary valve load, since the incremental rate for incremental loads on the primary valve of unit *B* is less than that for unit *A*. With both units operating at primary valve loads, the combined load is 120 megawatts. An increase in the combined load above this value could be supplied by the secondary valves of either or both units. By inspection of the secondary valve curves of Fig. 49, it is seen

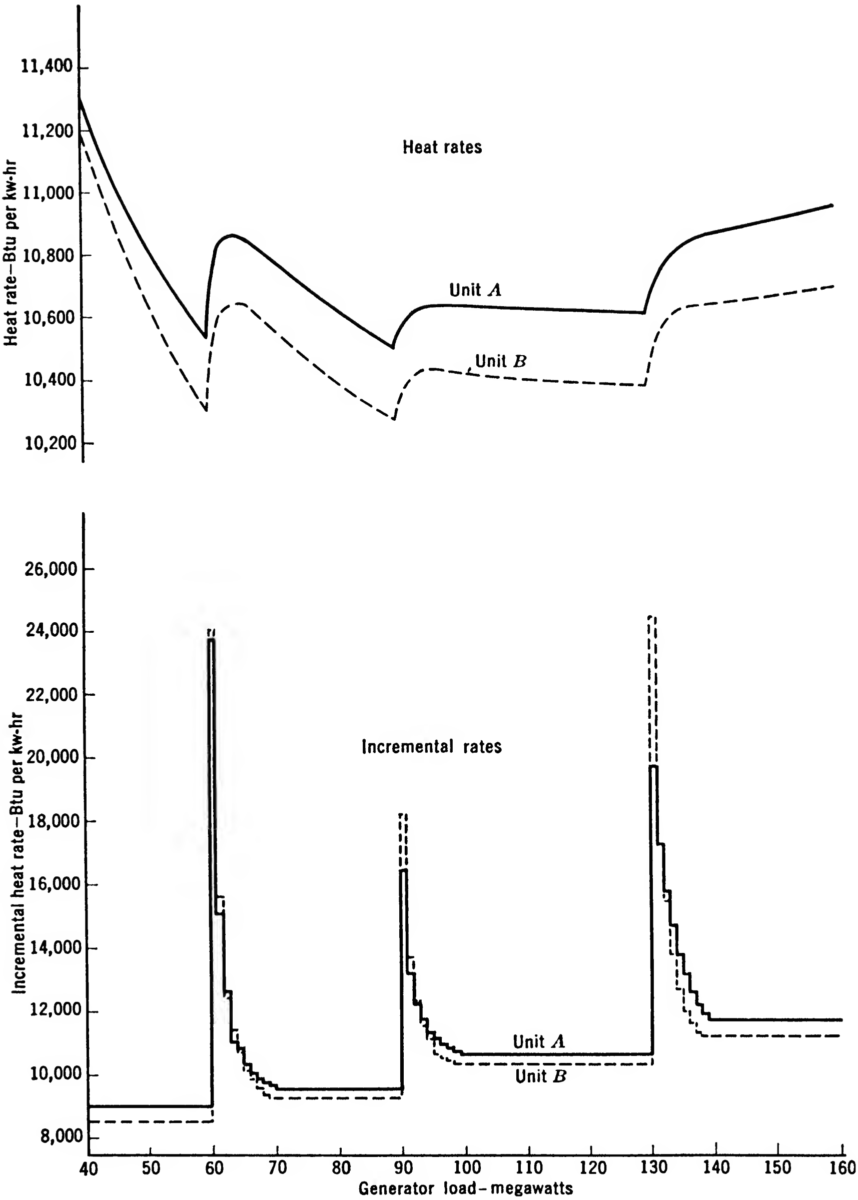


FIG. 48. Performance curves for two turbine-generators having peak incremental rate values.

that an increment of load up to 7.3 megawatts can be more efficiently generated on the secondary valve of unit A, while the reverse is true for increments of load greater than 7.3 megawatts. This merely means

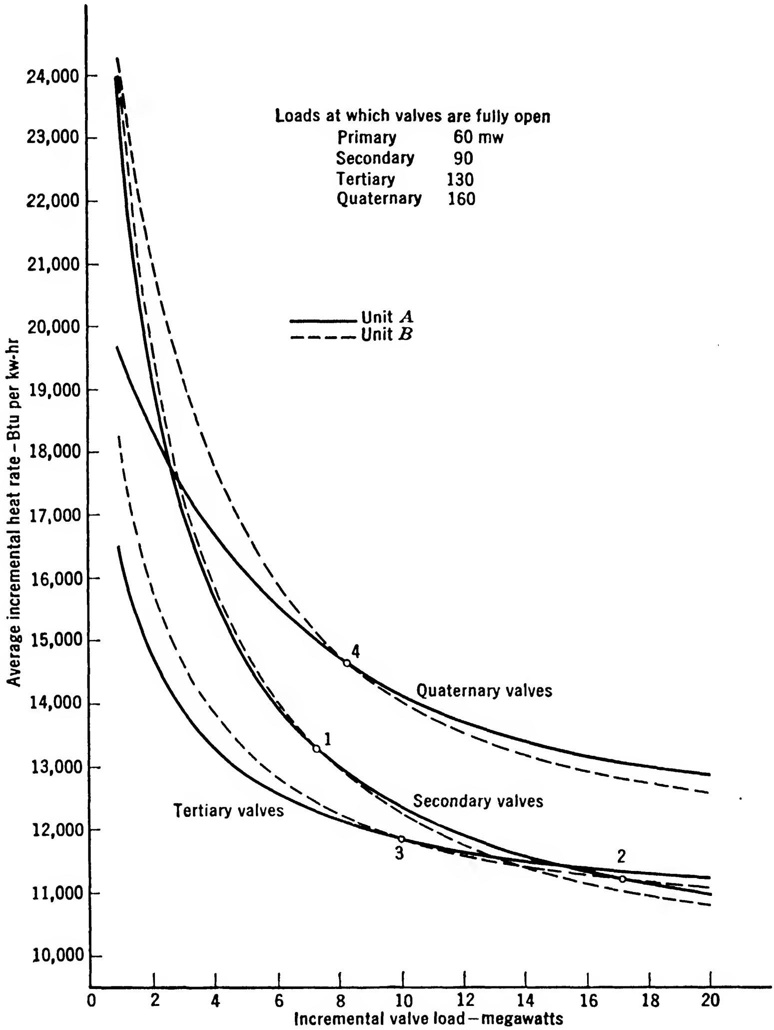


FIG. 49. Average incremental heat rates for steam valve loads derived from curves of Fig. 48.

that, for combined loads between 120 and 127.3 megawatts, unit B would be operated at a load of 60 megawatts while unit A would supply the remainder. For combined loads greater than 127.3 megawatts, unit A should be operated at 60 megawatts while unit B supplies the remain-

TABLE XIII

LOADING SEQUENCE FOR TWO UNITS OF FIG. 48

Loading Sequence	Load, megawatts			Increment Load Supplied by	
	Total	Unit *A*	Unit *B*	Unit	Valve
1	80 -100	40	40 - 60	*B*	Primary
2	100 -120	40 - 60	60	*A*	Primary
3	120 -127.3	60 - 67.3	60	*A*	Secondary
4	127.3-150	60	67.3- 90	*B*	Secondary
5	150 -167.1	60	90 -107.1	*B*	Tertiary
6	167.1-180	77.1- 90	90	*A*	Secondary
7	180 -190	90 -100	90	*A*	Tertiary
8	190 -220	90	90 -130	*B*	Tertiary
9	220 -260	90 -130	130	*A*	Tertiary
10	260 -268.3	130 -138.3	130	*A*	Quaternary
11	268.3-290	130	138.3-160	*B*	Quaternary
12	290 -320	130 -160	160	*A*	Quarternary

der until its secondary valve is fully open. The transition point corresponding to the increment of 7.3 megawatts is determined by the intersection of the two curves in question at point 1, which governs the sequence for loading the secondary valves of the units. Similarly, point 2 governs the sequence for loading the secondary valve of unit A and the tertiary valve of unit B. Points 3 and 4 apply to the sequence of loading the tertiary and quaternary valves, respectively.

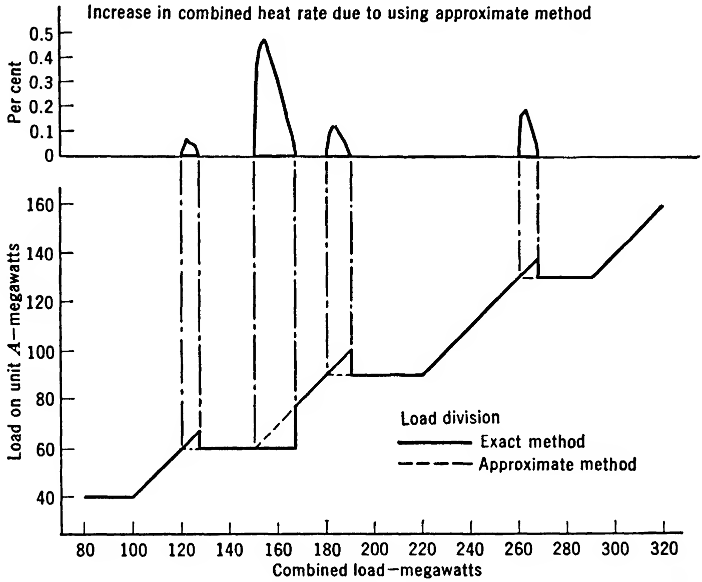


FIG. 50. Comparison between *exact* and *approximate* methods of loading the two units of Fig. 48.

The method employing the curves of Fig. 49 will be referred to as the "exact" method. The complete loading sequence for the two units in question, obtained by the application of this method, is shown in Table XIII. There are two practical objections to this method. First, it becomes increasingly cumbersome to apply with increase in the number of units. Second, a loading schedule is established requiring the shifting of load from one unit to another which is generally considered to be poor operating practice.

A simpler method is available which will give results within acceptable engineering accuracy. This will be referred to as the "approximate" method by which the loading of units is obtained from incremental rates established as average values between successive valve loads. Calculation of the average incremental rates for the two units is shown in Table XIV.

TABLE XIV

CALCULATION OF AVERAGE INCREMENTAL RATES FOR TURBINE-GENERATORS

Valve	Load Mw	Incremental Load Mw	Unit A Heat Rate Btu/Kw-hr	Unit A Input 10^6 Btu/Hr	Unit A Incremental Input 10^6 Btu/Hr	Unit A Incremental Heat Rate Btu/Kw-hr	Unit B Heat Rate Btu/Kw-hr	Unit B Input 10^6 Btu/Hr	Unit B Incremental Input 10^6 Btu/Hr	Unit B Incremental Heat Rate Btu/Kw-hr
	40		11,300	452			11,200	448		
Primary		20			180	9,000			170	8,500
	60		10,567	632			10,300	618		
Secondary		30			313	10,433			306	10,200
	90		10,500	945			10,267	924		
Tertiary		40			433	10,825			424	10,600
	130		10,600	1,378			10,369	1,348		
Quaternary		30			371	12,367			360	12,000
	160		10,931	1,749			10,675	1,708		

Figure 50 shows the variation in the loading of the units and in the combined input resulting from the use of the approximate method. Variation in the combined input occurs when there is a variation in the loading of the units, and this is limited to the range of loads corresponding to those portions of the turbine-generator input-output curves which can be established with the least accuracy. A specific advantage of the approximate over the exact method is the elimination of the need to draw the incremental rate curves for the individual turbine-generators. This follows from the fact that the approximate method will indicate that all but one unit should be operated at valve loads. The relative values of the average incremental rates will indicate the loading sequence, so that the problem of loading any number of units can be readily solved by means of a relatively simple tabulation in the manner to be discussed later in this chapter. Justification for the use of the approximate method is based on the simplicity of its application, especially when a large group of units is involved, and on the fact that any resulting loss in operating efficiency is more apparent than real.

Methods of Loading Turbine-Generators

Although the incremental loading of turbine-generators will always result in the maximum overall operating efficiencies, other methods of loading have been and still are in use. It is of interest to consider some of them. For the purpose of illustration, three turbine-generator units whose performance curves are shown in Fig. 51 will be used.

Base Loading to Capacity. The units are successively loaded to capacity in the order of their efficiencies. For the three units of Fig. 51 the sequence of loading each to capacity is C, B, and A, as shown in Table XV. This method of loading is based upon the assumption that the overall efficiency for a group of units will be highest if all but the most efficient unit are operated at minimum loads until the most efficient one is loaded to capacity. This assumption is obviously not correct unless the load division resulting from this method is the same as that obtained by the incremental loading of the units.

Base Loading to Most Efficient Load. The units are successively loaded, in ascending order of their heat rates, to their most efficient loads. When all units are operating at their most efficient loads, they are loaded to capacity in the same order. Thus, as shown in Table XVI, the units are first loaded to their most efficient loads in the order C, B, and A, and then loaded to capacity in the same order.

Proportional to Capacity. The loads on the units are in proportion to their rated capacity. Referring to Table XVII it will be seen that the

loads on units A, B, and C are in the ratio 40 : 60 : 100, respectively, corresponding to the rated capacities of the units.

Proportional to Most Efficient Load. The loads on the units are kept in proportion to their most efficient loads. After the units have reached

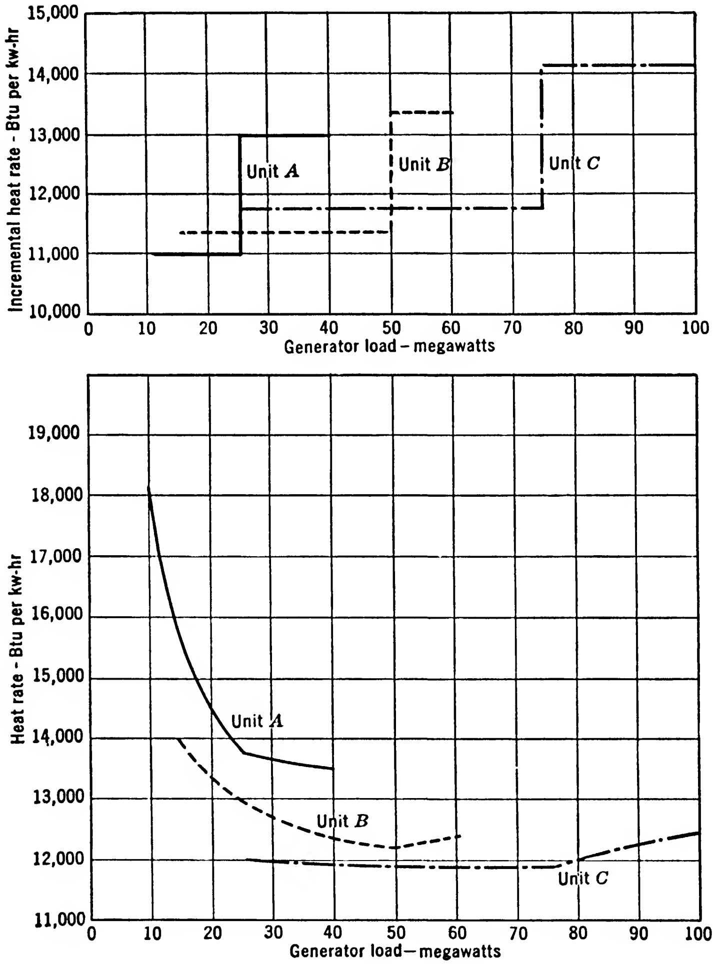


FIG. 51. Performance curves of three turbine-generator units.

their most efficient loads, the additional load on each is made proportional to the difference between the rated capacity and the most efficient load. Application of this method is illustrated in Table XVIII. The most efficient loads for units A, B, and C are 25, 50, and 75 megawatts, respectively. Hence the load on each unit would be in the ratio of

TABLE XV

BASE LOADING TO CAPACITY

Load, Megawatts				Input, 10^6 Btu per Hour				Combined Heat Rate Btu/Kw-Hr
Unit *A*	Unit *B*	Unit *C*	Total	Unit *A*	Unit *B*	Unit *C*	Total	
10	15	25	50	180	210	300	690	13,800
10	15	35	60	180	210	418	808	13,470
10	15	45	70	180	210	536	926	13,230
10	15	55	80	180	210	654	1,044	13,050
10	15	65	90	180	210	772	1,162	12,910
10	15	75	100	180	210	890	1,280	12,800
10	15	80	105	180	210	961	1,351	12,870
10	15	90	115	180	210	1,103	1,493	12,980
10	15	100	125	180	210	1,245	1,635	13,080
10	25	100	135	180	324	1,245	1,749	12,960
10	35	100	145	180	438	1,245	1,863	12,850
10	45	100	155	180	552	1,245	1,977	12,760
10	50	100	160	180	609	1,245	2,034	12,710
10	55	100	165	180	676	1,245	2,101	12,730
10	60	100	170	18C	743	1,245	2,168	12,750
15	60	100	175	235	743	1,245	2,223	12,700
20	60	100	180	290	743	1,245	2,278	12,660
25	60	100	185	345	743	1,245	2,333	12,610
30	60	100	190	410	743	1,245	2,398	12,620
35	60	100	195	475	743	1,245	2,463	12,630
40	60	100	200	540	743	1,245	2,528	12,640

TABLE XVI

BASE LOADING TO MOST EFFICIENT LOAD

Load, Megawatts				Input, 10^6 Btu per Hour				Combined Heat Rate Btu/Kw-hr
Unit *A*	Unit *B*	Unit *C*	Total	Unit *A*	Unit *B*	Unit *C*	Total	
10	15	25	50	180	210	300	690	13,800
10	15	35	60	180	210	418	808	13,470
10	15	45	70	180	210	536	926	13,230
10	15	55	80	180	210	654	1,044	13,050
10	15	65	90	180	210	772	1,162	12,910
10	15	75	100	180	210	890	1,280	12,800
10	25	75	110	180	324	890	1,394	12,670
10	35	75	120	180	438	890	1,508	12,570
10	45	75	130	180	552	890	1,622	12,480
10	50	75	135	180	609	890	1,679	12,440
15	50	75	140	235	609	890	1,734	12,390
20	50	75	145	290	609	890	1,789	12,340
25	50	75	150	345	609	890	1,844	12,290
25	50	80	155	345	609	961	1,915	12,350
25	50	90	165	345	609	1,103	2,057	12,470
25	50	100	175	345	609	1,245	2,199	12,570
25	55	100	180	345	676	1,245	2,266	12,590
25	60	100	185	345	743	1,245	2,333	12,610
30	60	100	190	410	743	1,245	2,398	12,620
35	60	100	195	475	743	1,245	2,463	12,630
40	60	100	200	540	743	1,245	2,528	12,640

TABLE XVII

LOADING PROPORTIONAL TO CAPACITY

Load, Megawatts				Input, 10^6 Btu per Hour				Combined Heat Rate Btu/Kw-hr
Unit *A*	Unit *B*	Unit *C*	Total	Unit *A*	Unit *B*	Unit *C*	Total	
10	15	25	50	180	210	300	690	13,800
12	18	30	60	202	244.2	359	805.2	13,420
14	21	35	70	224	278.4	418	920.4	13,150
16	24	40	80	246	312.6	477	1,035.6	12,950
18	27	45	90	268	346.8	536	1,150.8	12,790
20	30	50	100	290	381	595	1,266	12,660
22	33	55	110	312	415.2	654	1,381.2	12,560
24	36	60	120	334	449.4	713	1,496.4	12,470
25	37.5	62.5	125	345	466.5	742.5	1,554	12,430
26	39	65	130	358	483.6	772	1,613.6	12,410
28	42	70	140	384	517.8	831	1,732.8	12,380
30	45	75	150	410	552.0	890	1,852	12,350
32	48	80	160	436	586.2	961	1,983.2	12,400
33.3	50	83.3	166.6	452.9	609.0	1,007.9	2,069.8	12,420
34	51	85	170	462	622.4	1,032	2,116.4	12,450
36	54	90	180	488	662.6	1,103	2,253.6	12,530
38	57	95	190	514	702.8	1,174	2,390.8	12,580
40	60	100	200	540	743	1,245	2,528.0	12,640

TABLE XVIII

LOADING PROPORTIONAL TO MOST EFFICIENT LOAD

Load, Megawatts				Input, 10^6 Btu per Hour				Combined Heat Rate Btu/Kw-hr
Unit *A*	Unit *B*	Unit *C*	Total	Unit *A*	Unit *B*	Unit *C*	Total	
10	15	25	50	180	210	300	690	13,800
10	18	27	55	180	244.2	323.6	747.8	13,600
10	20	30	60	180	267	359	806.0	13,430
12	24	36	72	202	312.6	429.8	944.4	13,120
14	28	42	84	224	358.2	500.6	1,082.8	12,890
16	32	48	96	246	403.8	571.4	1,221.2	12,720
18	36	54	108	268	449.4	642.2	1,359.6	12,590
20	40	60	120	290	495	713	1,498.0	12,480
22	44	66	132	312	540.6	783.8	1,636.4	12,400
24	48	72	144	334	586.2	854.6	1,774.8	12,330
25	50	75	150	345	609	890	1,844	12,290
28	52	80	160	384	635.8	961	1,980.8	12,380
31	54	85	170	423	662.6	1,032	2,117.6	12,460
34	56	90	180	462	689.4	1,103	2,254.4	12,520
37	58	95	190	501	716.2	1,174	2,391.2	12,590
40	60	100	200	540	743	1,245	2,528	12,640

1 : 2 : 3 up to and including a total load of 150 megawatts. The capacity above the most efficient loads are 15, 10, and 25 megawatts, respectively, or in the ratio of 3 : 2 : 5, so that, for total loads in excess of 150 megawatts, the load on each unit would be as follows:

Unit	Megawatts
A	$25 + 0.3\ (L - 150)$
B	$50 + 0.2\ (L - 150)$
C	$75 + 0.5\ (L - 150)$

where L is the total load.

Incremental. The load is divided to keep the units operating at equal incremental rates. The incremental loading of the units is illustrated in

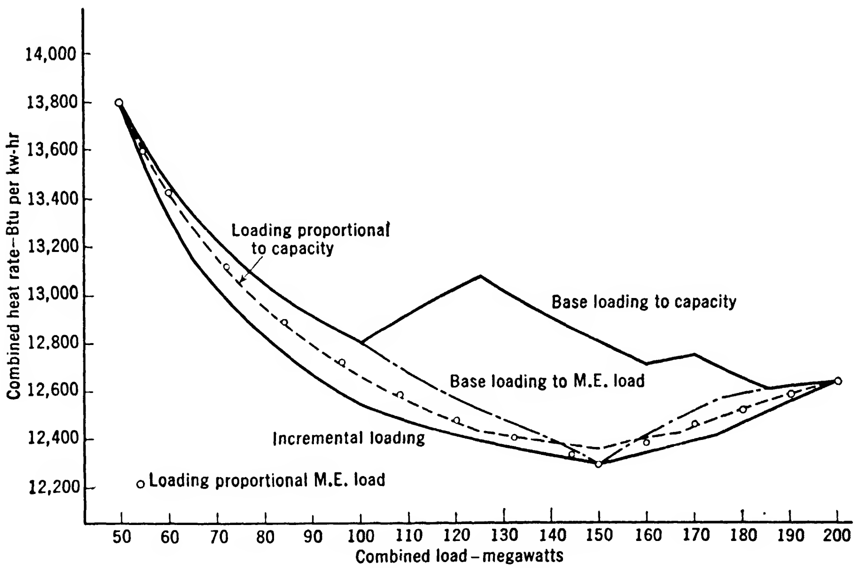


Fig. 52. Effect of method of loading on the combined heat rate of the units of Fig. 51.

Table XIX. By inspection of the incremental rate curves of Fig. 51, it is obvious that the units should first be loaded to their most efficient loads in the order A, B, and C, and then to capacity in the same order.

Figure 52 shows the combined heat rate curves obtained for the three units by the application of the different methods of loading. The incremental loading results in the lowest overall heat rates, which are not equaled by any of the other methods except when the load division is the same. Although the two methods based upon proportional loading will rarely give the best distribution, they are reasonably satisfactory for

TABLE XIX

INCREMENTAL LOADING

Load, Megawatts				Input, 10^6 Btu per Hour				Combined Heat Rate Btu/Kw-hr
Unit *A*	Unit *B*	Unit *C*	Total	Unit *A*	Unit *B*	Unit *C*	Total	
10	15	25	50	180	210	300	690	13,800
15	15	25	55	235	210	300	745	13,550
20	15	25	60	290	210	300	800	13,330
25	15	25	65	345	210	300	855	13,150
25	25	25	75	345	324	300	969	12,920
25	35	25	85	345	438	300	1,083	12,740
25	45	25	95	345	552	300	1,197	12,600
25	50	25	100	345	609	300	1,254	12,540
25	50	35	110	345	609	418	1,372	12,470
25	50	45	120	345	609	536	1,490	12,420
25	50	55	130	345	609	654	1,608	12,370
25	50	65	140	345	609	772	1,726	12,330
25	50	75	150	345	609	890	1,844	12,290
30	50	75	155	410	609	890	1,909	12,320
35	50	75	160	475	609	890	1,974	12,340
40	50	75	165	540	609	890	2,039	12,360
40	55	75	170	540	676	890	2,106	12,390
40	60	75	175	540	743	890	2,173	12,420
40	60	80	180	540	743	961	2,244	12,470
40	60	90	190	540	743	1,103	2,386	12,560
40	60	100	200	540	743	1,245	2,528	12,640

quick approximations when detailed performance data are not available. The base loading methods show up unfavorably in this illustration. Under some circumstances they would come much closer to the results obtained with incremental loading. In general, if enough data are available to permit the satisfactory application of these methods, there are enough for the application of incremental rates by which the correct results can be obtained.

Incremental Loading Applied

The application of incremental loading to a group of turbine-generators whose heat rate curves are shown in Fig. 53 will now be demon-

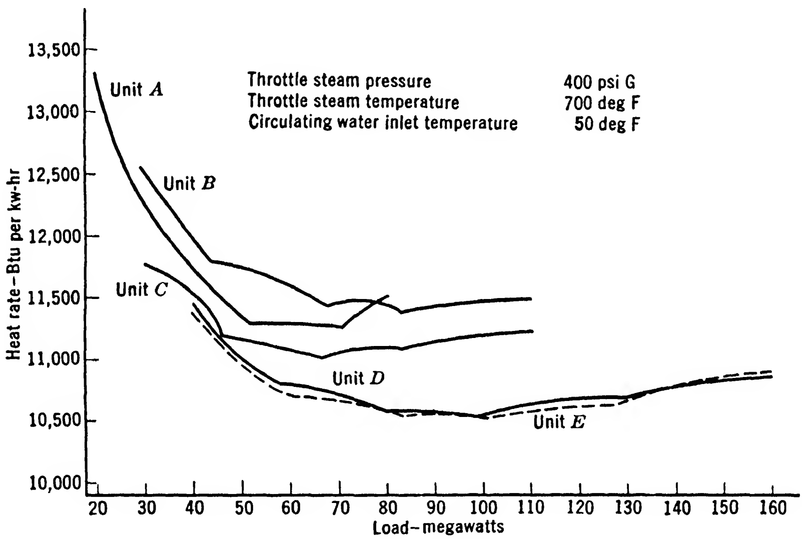


FIG. 53. Heat rate curves for five turbine-generator units used to illustrate application of incremental loading.

strated. From the curves of Fig. 53 incremental rate values can be established for each unit as averages between successive valve loads, which are indicated by the break points on the heat rate curves. The average incremental heat rates for the units, computed by the method indicated in Table XIV, are shown in Table XX. By arranging the tabulated data in ascending order of average incremental heat rate values, the loading sequence for the units is established as shown in Table XXI, the key table from which the loading sequence for any combination of units can be determined.

Suppose, for example, that it is desired to establish a loading schedule when units A, C, and E only are on the bus. The loading sequence for

TABLE XX

TURBINE-GENERATOR AVERAGE INCREMENTAL RATES

Unit	Valve	Load Range Megawatts	Average Incremental Heat Rate Btu/Kw-hr
A	Primary	20– 52	10,015
	Secondary	52– 71	11,243
	Tertiary	71– 80	13,492
B	Primary	30– 44	10,437
	Secondary	44– 68	10,791
	Tertiary	68– 83	11,226
	Quaternary	83–110	11,807
C	Primary	30– 46	10,233
	Secondary	46– 67	10,622
	Tertiary	67– 83	11,381
	Quaternary	83–110	11,635
D	Primary	40– 58	9,281
	Secondary	58– 80	10,009
	Tertiary	80– 99	10,396
	Quaternary	99–129	11,161
	Quinary	129–160	11,454
E	Primary	40– 62	9,445
	Secondary	62– 84	10,156
	Tertiary	84–101	10,466
	Quaternary	101–128	10,985
	Quinary	128–160	11,726

TABLE XXI

TURBINE-GENERATOR LOADING SEQUENCE

Unit	Valve	Load Range Megawatts	Average Incremental Heat Rate Btu/Kw-hr
D	Primary	40– 58	9,281
E	Primary	40– 62	9,445
D	Secondary	58– 80	10,009
A	Primary	20– 52	10,015
E	Secondary	62– 84	10,156
C	Primary	30– 46	10,233
D	Tertiary	80– 99	10,396
B	Primary	30– 44	10,437
E	Tertiary	84–101	10,466
C	Secondary	46– 67	10,622
B	Secondary	44– 68	10,791
E	Quaternary	101–128	10,985
D	Quaternary	99–129	11,161
B	Tertiary	68– 83	11,226
A	Secondary	52– 71	11,243
C	Tertiary	67– 83	11,381
D	Quinary	129–160	11,454
C	Quaternary	83–110	11,635
E	Quinary	128–160	11,726
B	Quaternary	83–110	11,807
A	Tertiary	71– 80	13,492

these units is obtained from the sequence shown in Table XXI with units *B* and *D* omitted, resulting in the loading schedule shown in Table XXII. Referring to this table, the loads for the individual units are tabulated first. The total load is then obtained by addition of the individual unit loads.

TABLE XXII

Loading Schedule for Three Units

Load, Megawatts				Average Incremental Heat Rate Btu/Kw-hr
Total	Unit *A*	Unit *C*	Unit *E*	
90–112	20	30	40– 62	9,445
112–144	20–52	30	62	10,015
144–166	52	30	62– 84	10,156
166–182	52	30– 46	84	10,233
182–199	52	46	84–101	10,466
199–220	52	46– 67	101	10,622
220–247	52	67	101–128	10,985
247–266	52–71	67	128	11,243
266–282	71	67– 83	128	11,381
282–309	71	83–110	128	11,635
309–341	71	110	128–160	11,726
341–350	71–80	110	160	13,492

With the loading sequence established on the basis of all units being on the bus, as illustrated by Table XXI, a loading schedule can be prepared for any combination of units in service in a few minutes.

It has previously been shown, by reference to Fig. 36, that, with constant throttle flow, a change in the temperature of the inlet circulating water causes changes in the load that can be generated and in the heat rate of the unit. For the preparation of loading schedules to take into account the inlet temperature of the circulating water, it is necessary to establish, for each turbine-generator unit, the load and heat rate as a function of the inlet circulating water temperature at constant throttle steam flow. This condition can be satisfied by establishing for each unit in the plant a set of curves similar to those of Fig. 36.

Since the loading of the units is based on incremental rates which are average values between successive valve loads, consideration of the effect of inlet circulating water temperature may be limited to the condition when the steam admission valves are fully open. To simplify the derivation of the loading schedules it is only necessary to establish the valve loads and the turbine-generator heat rates corresponding thereto as functions of the inlet circulating water temperature. This is illustrated for a unit with five steam admission valves by the curves of Figs. 54 and 55. The average incremental heat rates derived from these curves, by using the method illustrated in Table XIV, are shown in Fig. 56. Curves like

these, when established for each unit in the plant, will permit the calculation of a loading schedule for any combination of units and for any inlet circulating water temperature.

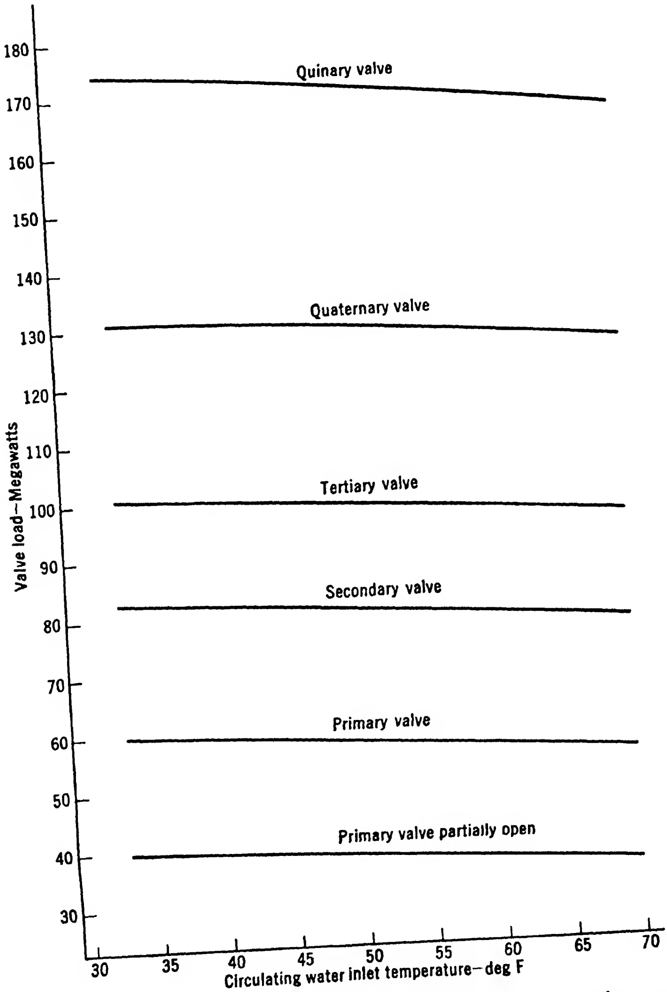


FIG. 54. Relation between circulating water inlet temperature and loads on generator with steam admission valves fully open.

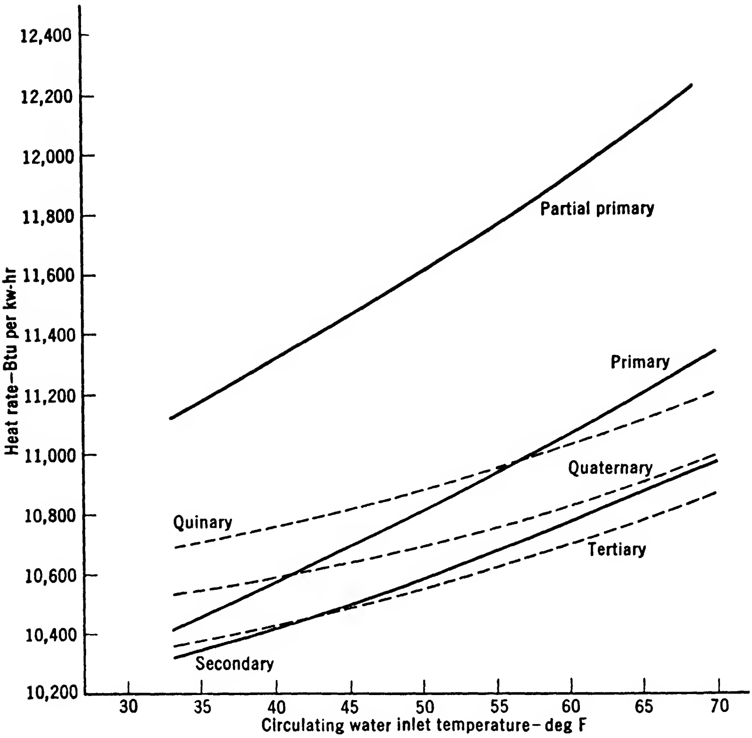


FIG. 55. Relation between circulating water inlet temperature and heat rates corresponding to the valve loads shown in Fig. 54.

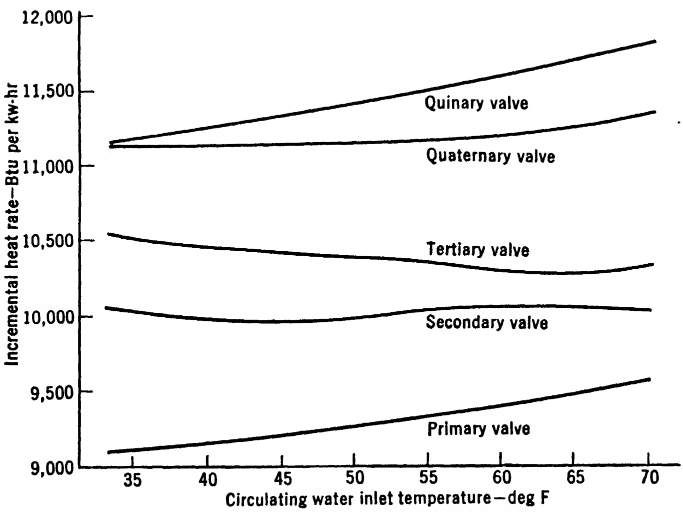


FIG. 56. Relation between circulating water inlet temperature and average incremental heat rates derived from Figs. 54 and 55.

Sequence of Adding Turbine-Generators

Incremental rates are applied to determine the division of load among two or more units supplying a common load. This presupposes that the particular units to supply the load have been selected and tied to the station buses, and it raises the problem of deciding in what order and at what station loads the units should be put in or removed from service. In this connection it cannot be too strongly stressed that the incremental rates of the respective units have no direct influence on the sequence of adding units to the bus. The problem of which unit to put on the line must be solved by the heat rates rather than by the incremental rates of the units. It is only after the particular units are on the line that the incremental rates become of greater importance and determine the proper loading.

By plotting the input-output or heat rate curves of all units to a common scale, the sequence for putting the units in service can generally be established by inspection. Referring to the curves of Fig. 53, it will be seen that the order in which these units should be put on the line is *E*, *D*, *C*, *A*, and *B*, this being the order of ascending heat rates. The sequence for adding the units to the line becomes more involved if the heat rate curves of the respective units intersect so that one unit is more efficient at loads less than, and the other at loads greater than, that corresponding to the point of intersection. In such a case the choice will be governed by the load cycle. The principle involved is illustrated by the following example.

Example. A station has a load of 20,000 kilowatts for 3 hours and a load of 40,000 kilowatts for 2 hours. Two units are available, and only one is required to supply the load. The heat rates of each unit for the above loads are as follows:

Load, kilowatts	Heat Rate—Btu per kw-hr Unit *A*	Unit *B*
20,000	15,000	14,000
40,000	14,000	14,500

Which unit should be operated?

If unit *A* is operated the input will be

$$\begin{aligned} 20{,}000 \text{ kw} \times 15{,}000 \text{ Btu per kw-hr} \times 3 \text{ hours} &= 900 \times 10^6 \text{ Btu} \\ 40{,}000 \times 14{,}000 \times 2 &= 1{,}120 \\ \text{Total} &\quad 2{,}020 \times 10^6 \text{ Btu} \end{aligned}$$

If unit *B* is operated the input will be

$$\begin{aligned} 20{,}000 \text{ kw} \times 14{,}000 \text{ Btu per kw-hr} \times 3 \text{ hours} &= 840 \times 10^6 \text{ Btu} \\ 40{,}000 \times 14{,}500 \times 2 &= 1{,}160 \\ \text{Total} &\quad 2{,}000 \times 10^6 \text{ Btu} \end{aligned}$$

By operating unit *B* a saving of 20×10^6 Btu will be realized.

Determination of Most Economical Combination

The determination of the sequence for adding units to the line is merely a preliminary step in the solution of the major problem of determining the combination of units that will supply a particular load at the

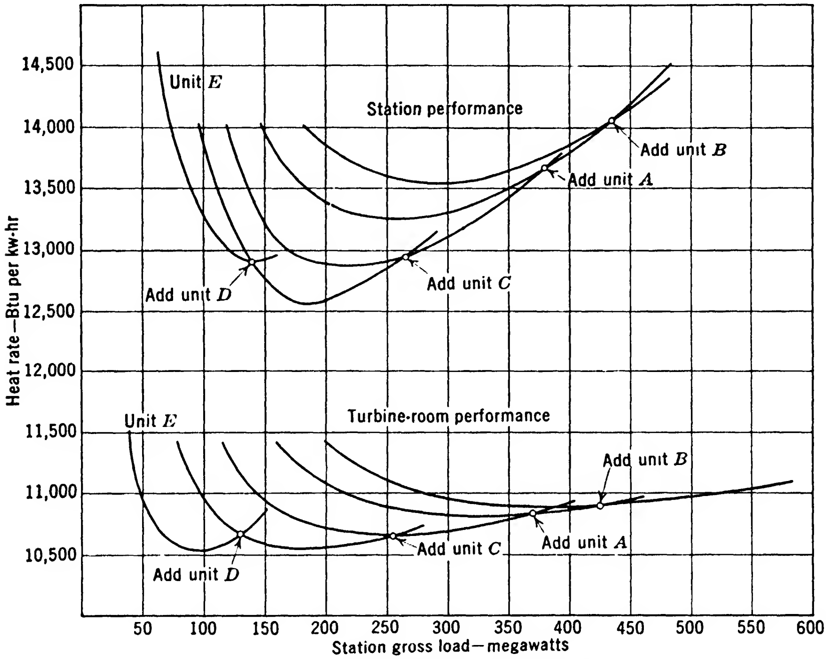


Fig. 57. Proper sequence for putting the units of Fig. 53 on the line.

least overall heat rate. The number of possible combinations is a function of the number of units available for service. Thus in a station containing five units, thirty-one combinations are possible:

Number of Units in Operation	Number of Combinations
1	5
2	10
3	10
4	5
5	1
Total	31

Theoretically, for a complete analysis, thirty-one heat rate curves, one for each combination, would be required, so that, for any given load,

that combination would be selected which could supply the load at the lowest heat rate.

Practically, it is seldom necessary to compute the number of curves indicated. With the sequence for adding units established, heat rate curves for successive combinations of units should be plotted, as illustrated for the units of Fig. 53 by the lower set of curves of Fig. 57. These show the overall heat rates for combinations from one to five units, using the sequence previously established. The loads at which to add units are then determined by the points of intersection, so that, for the units in question, a second unit should be added at a load of 130 megawatts, a third unit at 255 megawatts, a fourth unit at 370 megawatts, and the fifth unit at a load of 425 megawatts.

When one or more units are unavailable for service the procedure should be repeated, using the performance data for the available units. In computing the overall heat rate curve for any combination of units, the total load should be divided incrementally among the individual units.

Effect of Auxiliary Power Consumption

In discussing the problem thus far no consideration has been given to the effect of auxiliary power consumption. It is obvious that the addition of a unit will result in increased auxiliary power consumption and, therefore, will increase the total generation required for the given bus load. The net effect is generally to extend the load at which it becomes economical to add the unit. This is illustrated by the upper set of curves in Fig. 57, which shows the relation between the overall station net heat rate and the gross station load for the same combinations of units for which the lower set of curves was computed.

For all practical purposes the problem may be treated with regard to the turbine-room economy only. To treat it with regard to the overall station economy would require numerous calculations depending upon the number of boiler combinations involved and the ability to segregate the turbine auxiliary requirements from those of the rest of the station.

In Table XXIII are shown typical loading schedules for combinations of two or more units, derived from the loading sequence indicated in Table XXI. The most economical combinations are shown by the portions of the schedules between the horizontal lines.

Effect of Reserve Requirements

System reserve requirements may limit the extent to which the most economical combination of units can be operated to supply a given load,

in that more capacity may have to be operated on the bus than is required for economy. The effect of providing reserve capacity may therefore be reflected in lower operating efficiency of the plant. This is illustrated for an actual system in Fig. 58, which shows by means of the

TABLE XXIII

Typical Turbine-Generator Loading Schedules

Station Load	2 Units		3 Units			4 Units				5 Units				
	D	*E*	*C*	*D*	*E*	*A*	*C*	*D*	*E*	*A*	*B*	*C*	*D*	*E*
80	40	40												
100	58	42												
120	58	62	30	50	40									
140	78	62	30	58	52	20	30	50	40					
160	80	80	30	68	62	20	30	58	52	20	30	30	40	40
180	96	84	30	80	70	20	30	68	62	20	30	30	58	42
200	99	101	36	80	84	28	30	80	62	20	30	30	58	62
220	99	121	46	80	94	48	30	80	62	20	30	30	78	62
240	112	128	49	80	101	52	30	80	78	38	30	30	80	62
260	132	128	67	80	113	52	44	80	84	52	30	30	80	68
265	137	128	67	80	118	52	46	83	84	52	30	30	80	73
280	152	128	67	85	128	52	46	98	84	52	30	34	80	84
300	160	140	67	105	128	52	48	99	101	52	30	46	88	84
320	160	160	67	125	128	52	67	99	102	52	39	46	99	84
340			83	129	128	52	67	99	122	52	44	46	99	99
360			83	149	128	52	67	113	128	52	44	64	99	101
380			92	160	128	56	67	129	128	52	61	67	99	101
400			110	160	130	71	72	129	128	52	68	67	99	114
420			110	160	150	71	83	138	128	52	68	67	105	128
435						71	83	153	128	52	68	67	120	128
440						71	83	158	128	52	68	67	125	128
460						71	101	160	128	53	83	67	129	128
480						71	110	160	139	71	83	69	129	128
500						71	110	160	159	71	83	83	135	128
520										71	83	83	155	128
540										71	83	98	160	128
560										71	83	110	160	136
580										71	83	110	160	156
600										71	99	110	160	160
620										80	110	110	160	160

All loads in megawatts.

cross-hatched areas the capacity operated in excess of that required for best economy and the loss in efficiency resulting therefrom.

Effect of Starting Losses

Operators of a station supplying a variable load similar to that shown in Fig. 58 are frequently confronted with the problem of deciding whether or not it would be economical to shut down one or more units during relatively short periods of time. If a unit is shut down, it would have to

be put back on the bus at the end of the period, and this would usually involve losses incidental to restoring the unit to service. During the period of shutdown, it would be good practice to have the turbine spindle rotating at very low speeds in order to prevent unequal temperature distribution within the turbine casing. In many stations, especially where machines of large capacity are involved, this is accomplished by means of a motor-driven turning gear connected to the turbine shaft. Where

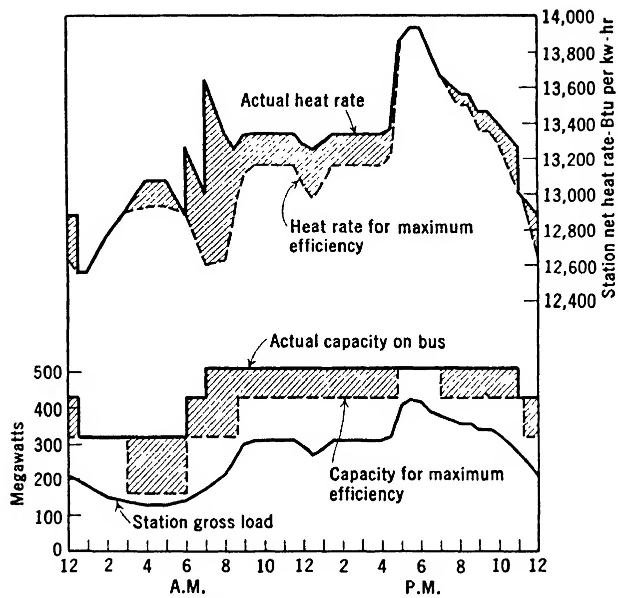


FIG. 58. Effect of turbine-generator reserve requirements on overall station heat rate.

this mechanism is not provided the slow rotation is accomplished by the admission of a small amount of steam to the turbine. The condensate of the steam used to rotate the spindle and to restore the unit to service by bringing it up to speed will frequently be thrown away if there is any possibility of contamination with the condenser cooling water, as might happen if salt water is used as the cooling medium.

Thermally, the starting loss would be represented by the heat content of the steam used for the above purposes. Other losses would involve the cost of fresh water to replace the condensate thrown overboard and the cost of treating the make-up water with chemicals to make it suitable for boiler feedwater.

To illustrate, consider the period from 3:00 A.M. to 6:00 A.M. in Fig. 58, during which the load could be supplied by one unit instead of two. The

proper operating practice can be determined by calculating the heat input to the station under both conditions as shown below.

			Two-Unit Operation		One-Unit Operation	
	Load in Megawatts		Station Net Heat Rate	Station Input	Station Net Heat Rate	Station Input
Time	Gross	Net	Btu per kw-hr	10^6 Btu per hr	Btu per kw-hr	10^6 Btu per hr
3 A.M.	140	134.0	12,900	1,728.6	12,900	1,728.6
4	130	124.5	13,070	1,627.2	12,930	1,609.8
5	130	124.5	13,070	1,627.2	12,930	1,609.8
6	140	134.0	12,900	1,728.6	12,900	1,728.6
Integrated total				4,983.0		4,948.2
Starting losses				0		100.0
Total station input				4,983.0		5,048.2

The above calculation clearly indicates that two units should be operated during the period.

CHAPTER IV

STATION PERFORMANCE

Independent Loading of Boilers and Turbines

In the preceding two chapters consideration was given to the problem of load division in the boiler and turbine rooms. The theoretical aspects and the extent to which they could be modified in the interest of simplification were indicated. It is to be noted that the problems of load division in the boiler and turbine rooms were considered as being independent of each other. That this procedure is permissible is demonstrated mathematically as follows:

Consider two boilers operating with two turbines; the performance characteristics of the boilers and of the turbines are different. Either turbine may be partly or wholly supplied from either boiler. With both boilers and both turbines on the line, let

L = total station load.
L_1 = load on turbine-generator 1.
L_2 = load on turbine-generator 2.
T_1 = heat input to turbine 1.
T_2 = heat input to turbine 2.
O_t = total heat output of both boilers available to the turbine room.
O_1 = heat output of boiler 1 available to the turbine room.
H = heat output of boiler 1 not used in turbine 1.
O_2 = heat output of boiler 2 available to the turbine room.
I_1 = heat input to boiler 1.
I_2 = heat input to boiler 2.
I_t = total heat input to both boilers.

It is assumed for the above that T_1 and T_2 include the inputs to the respective turbine-generator auxiliaries and that I_1 and I_2 are the inputs to the boilers necessary to furnish O_1 and O_2 and the inputs to the respective boiler auxiliaries. All station auxiliaries are assumed to be steam driven. Then

$$L = L_1 + L_2$$

$$T_1 + T_2 = O_t = O_1 + O_2$$

$$T_1 + H = O_1$$

$$T_2 - H = O_2$$

and

$$I_t = I_1 + I_2$$

Suppose that L_1 and O_1 are chosen as the two independent variables. The problem is then to determine, for any total load L, the values of L_1 and O_1 which will give the minimum station input. Assume that all the boiler and turbine input-output curves are continuous and that the respective incremental rates never decrease as the output is increased. Then the condition for a minimum is mathematically expressed by the equation

$$dI_t = 0 \qquad [44]$$

but

$$dI_t = \frac{\partial I_t}{\partial L_1} dL_1 + \frac{\partial I_t}{\partial O_1} dO_1 \qquad [45]$$

Since L_1 and O_1 are independent variables, it is necessary that

$$\frac{\partial I_t}{\partial L_1} = 0 \quad \text{and} \quad \frac{\partial I_t}{\partial O_1} = 0 \qquad [46]$$

in order that dI_t shall vanish. But

$$\frac{\partial I_t}{\partial O_1} = \frac{\partial(I_1 + I_2)}{\partial O_1} = \frac{\partial I_1}{\partial O_1} + \left(\frac{\partial I_2}{\partial O_2} \times \frac{\partial O_2}{\partial O_1}\right)$$

Also

$$O_2 = O_t - O_1$$

and

$$\frac{\partial O_2}{\partial O_1} = \frac{\partial O_t}{\partial O_1} - \frac{\partial O_1}{\partial O_1} = -1$$

since O_t is independent of O_1. Hence

$$\frac{\partial I_t}{\partial O_1} = \frac{\partial I_1}{\partial O_1} - \frac{\partial I_2}{\partial O_2} = 0$$

or

$$\frac{\partial I_1}{\partial O_1} = \frac{\partial I_2}{\partial O_2} \qquad [47]$$

This simply means that the boiler incremental rates are equal. Similarly

$$\frac{\partial I_t}{\partial L_1} = \frac{\partial I_1}{\partial L_1} + \left(\frac{\partial I_2}{\partial L_2} \times \frac{\partial L_2}{\partial L_1}\right) \tag{48}$$

or

$$\frac{\partial I_t}{\partial L_1} = \left(\frac{\partial I_1}{\partial O_1} \times \frac{\partial O_1}{\partial L_1}\right) + \left(\frac{\partial I_2}{\partial O_2} \times \frac{\partial O_2}{\partial L_2} \times \frac{\partial L_2}{\partial L_1}\right) \tag{49}$$

Also

$$\frac{\partial L_2}{\partial L_1} = \frac{\partial (L - L_1)}{\partial L_1} = \frac{\partial L}{\partial L_1} - \frac{\partial L_1}{\partial L_1} = -1 \tag{50}$$

Now, since from equation 46

$$\frac{\partial I_t}{\partial L_1} = 0 \tag{51}$$

substituting equations 50 and 51 in equation 49,

$$\frac{\partial I_1}{\partial O_1} \times \frac{\partial O_1}{\partial L_1} = \frac{\partial I_2}{\partial O_2} \times \frac{\partial O_2}{\partial L_2} \tag{52}$$

But from equation 47

$$\frac{\partial I_1}{\partial O_1} = \frac{\partial I_2}{\partial O_2}$$

Hence equation 52 reduces to

$$\frac{\partial O_1}{\partial L_1} = \frac{\partial O_2}{\partial L_2} \tag{53}$$

Furthermore, since $O_1 = T_1 + H$, and $O_2 = T_2 - H$,

$$\frac{\partial O_1}{\partial L_1} = \frac{\partial T_1}{\partial L_1} + \frac{\partial H}{\partial L_1} \tag{54}$$

and

$$\begin{aligned}\frac{\partial O_2}{\partial L_2} &= \frac{\partial T_2}{\partial L_2} - \frac{\partial H}{\partial L_2} \\ &= \frac{\partial T_2}{\partial L_2} - \left(\frac{\partial H}{\partial L_1} \times \frac{\partial L_1}{\partial L_2}\right) \\ &= \frac{\partial T_2}{\partial L_2} - \left[\frac{\partial H}{\partial L_1} \times \frac{\partial (L - L_2)}{\partial L_2}\right] \\ &= \frac{\partial T_2}{\partial L_2} - \left[\frac{\partial H}{\partial L_1} \times \left(\frac{\partial L}{\partial L_2} - \frac{\partial L_2}{\partial L_2}\right)\right]\end{aligned}$$

$$\frac{\partial O_2}{\partial L_2} = \frac{\partial T_2}{\partial L_2} + \frac{\partial H}{\partial L_1} \tag{55}$$

Substituting equations 54 and 55 in 53

$$\frac{\partial T_1}{\partial L_1} = \frac{\partial T_2}{\partial L_2} \qquad [56]$$

But $\partial T_1/\partial L_1$ and $\partial T_2/\partial L_2$ are the incremental heat rates of the two turbine-generators. Hence, the requirements for a minimum station input, as shown by equations 47 and 56, are to operate (*a*) the two boilers at the same incremental rates, and (*b*) the two turbine-generators at the same incremental rates, respectively. It has already been shown that conditions (*a*) and (*b*) result in minimum boiler-room and minimum turbine-room inputs, respectively.

Calculation of Station Performance Curves

The performance of the station is a function of the combined performance of the boiler and turbine rooms, and the accuracy with which the station performance can be computed therefore depends on the accuracy of the computed results for the boiler and turbine rooms. In this connection it is necessary to consider the purpose for computing the station performance. When a system consists of several interconnected generating stations, the allocation of load among the stations should be based upon the same principles as are applied in the boiler and turbine rooms. The requirement is that incremental rates be established for each station, raising the problem of how this may be done with a minimum amount of effort consistent with the desired accuracy.

At this point use can be made of the performance curves for the boiler and turbine rooms, which, when properly combined, will give the desired station performance. It should be kept in mind that, in the normal course of operation, a large number of curves will be necessary for the different possible combinations of turbines and boilers that may be operated in a particular station. For example, consider a station with an installation consisting of five turbine-generators and ten boilers so arranged that steam from any boiler can be supplied to the throttle of any turbine. Then, theoretically, there are 31 possible turbine-generator and 1,023 possible boiler combinations. For the station as a whole there is a total of 31,713 possible combinations of boilers and turbine-generators. Actually it is extremely improbable that computations for anywhere near this number of combinations would be necessary. However, the number of likely combinations would still be relatively large, so that the computations involved would be quite considerable.

With respect to load division among the generating stations of a system, the primary interest is to establish the incremental rate curves for the respective stations. This can be done without resorting to the use of

either input-output or heat rate curves for the station. The incremental rate for the station can be expressed as a function of the respective incremental rates for the boiler room, turbine room, and station auxiliaries,

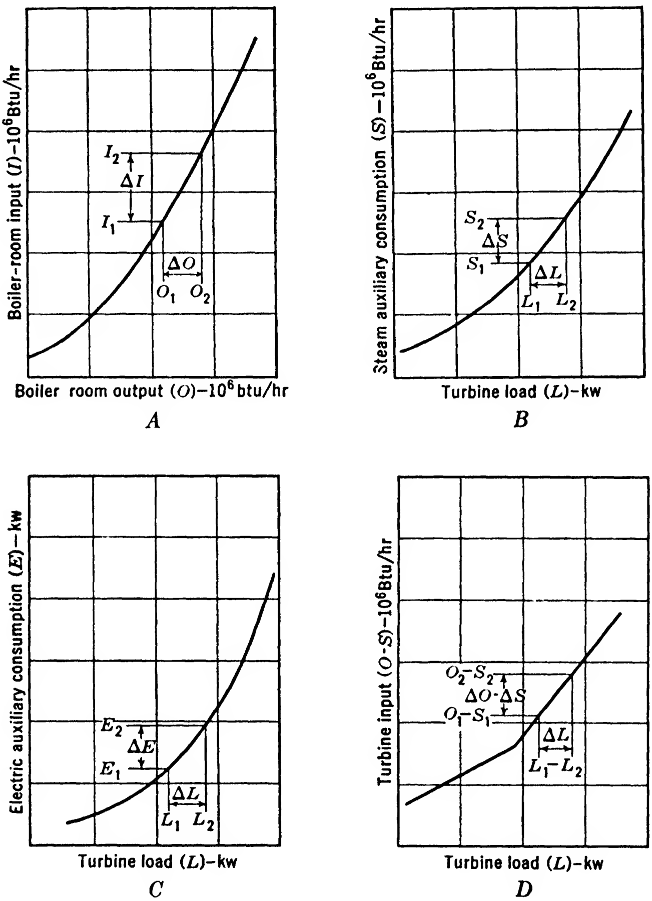


FIG. 59. Input-output curves used for the derivation of equation 63.

thus providing a simple method of calculation employing data which have previously been prepared for load division in the turbine and boiler rooms.

For the purpose of deriving the expression for the station incremental rate, consider the curves of Fig. 59.

Let L_1 and L_2 = two successive station loads, in kilowatts.

O_1 and O_2 = boiler outputs corresponding to the loads L_1 and L_2, respectively, in 10^6 Btu per hour.

I_1 and I_2 = the corresponding boiler inputs, in 10^6 Btu per hour.

S_1 and S_2 = the respective inputs to the steam-driven auxiliaries for the station, in 10^6 Btu per hour.

E_1 and E_2 = the respective electric auxiliary power consumptions of the station, in kilowatts.

Let R_b = the boiler-room incremental rate; then from Fig. 59*A*

$$R_b = \frac{\Delta I}{\Delta O} \qquad [57]$$

Let R_s = the station steam auxiliary incremental rate in Btu per kilowatt-hour; then from Fig. 59*B*

$$R_s = \frac{\Delta S}{\Delta L} \qquad [58]$$

Let R_e = the station electrical auxiliary incremental rate, a number; then from Fig. 59*C*

$$R_e = \frac{\Delta E}{\Delta L} \qquad [59]$$

Let R_t = the turbine-room incremental rate in Btu per kilowatt-hour; then from Fig. 59*D*

$$R_t = \frac{\Delta O - \Delta S}{\Delta L} = \frac{\Delta O}{\Delta L} - \frac{\Delta S}{\Delta L} = \frac{\Delta O}{\Delta L} - R_s \qquad [60]$$

Let R_{st} = the station *net* incremental rate in Btu per kilowatt-hour; then

$$R_{st} = \frac{I_2 - I_1}{(L_2 - E_2) - (L_1 - E_1)} = \frac{\Delta I}{\Delta L - \Delta E} \qquad [61]$$

but

$$\Delta I = R_b \times \Delta O$$

from equation 57, and

$$\Delta E = R_e \times \Delta L$$

from equation 59. Substituting in equation 61,

$$R_{st} = \frac{R_b \times \Delta O}{\Delta L - R_e \Delta L} = \frac{\Delta O}{\Delta L} \times \frac{R_b}{1 - R_e} \qquad [62]$$

but

$$\frac{\Delta O}{\Delta L} = R_t + R_s$$

from equation 60. Substituting in equation 62,

$$R_{st} = \frac{(R_t + R_s)R_b}{1 - R_e} \qquad [63]$$

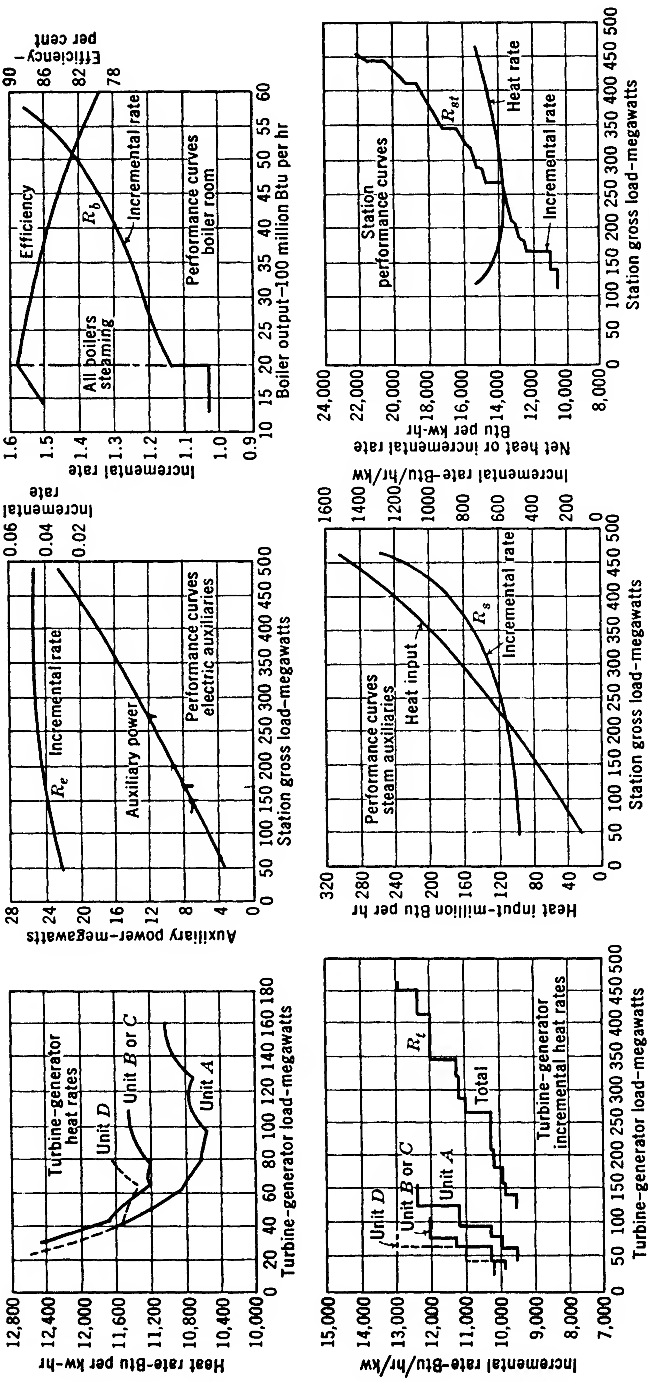


FIG. 60. Performance data used for the calculation of station incremental heat rates by equation 63.

When the station auxiliaries are essentially electrically driven, so that the consumption of the steam-driven auxiliaries is constant or not a function of the station load, equation 63 reduces to

$$R_{st} = \frac{R_t \times R_b}{1 - R_e} \tag{63a}$$

Conversely, if the station auxiliaries are essentially steam driven and the electric auxiliary consumption is not influenced by changes in the station load, equation 63 becomes

$$R_{st} = (R_t + R_s)R_b \tag{63b}$$

Equation 63 can be derived using differential calculus. Assuming that incremental loading has been independently applied to the turbines and boilers, so that the station performance represents continuous functions whose incremental rates are non-decreasing as the station load increases, let:

I_t = total station input.
O_t = total boiler output.
T = total turbine input.
S = heat input to steam auxiliaries.
E = power consumption of electric auxiliaries.
L = gross station load (total generator load).
L_n = net station load.

Then

$$O_t = T + S \qquad \text{and} \qquad L_n = L - E$$

so that

$$\frac{dI_t}{dL_n} = \frac{dI_t}{dO_t} \times \frac{d(T + S)}{dL} \times \frac{dL}{d(L - E)}$$

$$= \frac{dI_t}{dO_t} \times \left[\frac{dT}{dL} + \frac{dS}{dL}\right] \times \frac{1}{1 - \dfrac{dE}{dL}}$$

But

$$\frac{dI_t}{dO_t} = R_b; \ \frac{dT}{dL} = R_t; \ \frac{dS}{dL} = R_s; \ \frac{dE}{dL} = R_e; \ \frac{dI_t}{dL_n} = R_{st}$$

Hence

$$R_{st} = \frac{R_b(R_t + R_s)}{1 - R_e} \tag{63}$$

The application of the above equation is illustrated in Fig. 60 and Table XXIV for a given combination of turbines and boilers. In practice

TABLE XXIV

Sample Calculation of Station Performance Curves

Turbine-Generator Loads Megawatts					Electric Auxiliary Power Megawatts	Station Net Load Megawatts	Turbine Inputs, Million Btu/Hr				Steam Auxiliary Input, Million Btu/Hr	Total Boiler Room Output, Million Btu/Hr	R_t	R_b	R_e	R_s	R_{st}	Boiler Room Eff. Per Cent	Station Net Heat Rate Btu/Kw-hr
A	*B*	*C*	*D*	Total			*A*	*B*	*C*	*D*									
40	30	30	20	120	6.1	113.9	461	373	373	261	60	1,528		1.028	0.036	500	10,670	86.8	15,450
50	30	30	20	130	6.5	123.5	555	373	373	261	65	1,627	9,510	1.028	0.036	510	10,690	87.2	15,410
60	30	30	20	140	6.9	133.1	651	373	373	261	70	1,728		1.028	0.037	520	10,710	87.8	14,790
60	30	30	20	140	6.9	133.1	651	373	373	261	70	1,728		1.028	0.037	520	11,080	87.8	14,790
60	43	30	20	153	7.4	145.6	651	501	373	261	78	1,864	9,860	1.028	0.038	520	11,090	88.5	14,460
60	43	43	20	166	7.9	158.1	651	501	501	261	85	1,999		1.028	0.038	520	11,090	89.0	14,210
60	43	43	20	166	7.9	158.1	651	501	501	261	85	1,999		1.140	0.038	520	12,490	89.0	14,210
70	43	43	20	176	8.2	167.8	752	501	501	261	90	2,105	9,920	1.148	0.039	530	12,600	89.0	14,090
78	43	43	20	184	8.6	175.4	830	501	501	261	95	2,188		1.156	0.040	540	12,720	88.9	14,030
78	43	43	20	184	8.6	175.4	830	501	501	261	95	2,188		1.156	0.040	540	12,910	88.9	14,030
78	43	43	30	194	9.0	185.0	830	501	501	359	100	2,291	10,180	1.172	0.041	540	13,100	88.8	13,950
78	43	43	44	208	9.6	198.4	830	501	501	505	108	2,445		1.184	0.041	550	13,250	88.5	13,930
78	43	43	44	208	9.6	198.4	830	501	501	505	108	2,445		1.184	0.041	550	13,350	88.5	13,930
85	43	43	44	215	9.9	205.1	902	501	501	505	112	2,521	10,260	1.187	0.042	560	13,410	88.4	13,910
96	43	43	44	226	10.3	215.7	1,014	501	501	505	118	2,639		1.194	0.042	560	13,490	88.2	13,870
96	43	43	44	226	10.3	215.7	1,014	501	501	505	118	2,639		1.194	0.042	560	13,490	88.2	13,870
96	64	43	44	247	11.3	235.7	1,014	717	501	505	131	2,868	10,260	1.210	0.043	580	13,710	87.7	13,870
96	64	64	44	268	12.1	255.9	1,014	717	717	505	142	3,095		1.221	0.044	600	13,870	87.3	13,850

TABLE XXIV—*Continued*

SAMPLE CALCULATION OF STATION PERFORMANCE CURVES

Turbine-Generator Loads Megawatts					Electric Auxiliary Power Mega-watts	Station Net Load Mega-watts	Turbine Inputs, Million Btu/Hr				Steam Auxiliary Input, Million Btu/Hr	Total Boiler Room Output, Million Btu/Hr	R_t	R_b	R_e	R_s	R_{st}	Boiler Room Eff. Per Cent	Station Net Heat Rate Btu/Kw-hr
A	*B*	*C*	*D*	Total			*A*	*B*	*C*	*D*									
96	64	64	44	268	12.1	255.9	1,014	717	717	505	142	3,095		1.221	0.044	600	14,800	87.3	13,850
96	64	64	55	279	12.5	266.5	1,014	717	717	629	149	3,226	10,990	1.230	0.045	620	14,950	87.1	13,900
96	64	64	64	288	12.9	275.1	1,014	717	717	725	155	3,328		1.236	0.045	630	15,040	86.9	13,920
96	64	64	64	288	12.9	275.1	1,014	717	717	725	155	3,328		1.236	0.045	630	15,300	86.9	13,920
110	64	64	64	302	13.6	288.4	1,182	717	717	725	164	3,505	11,190	1.250	0.045	650	15,500	86.5	14,050
128	64	64	64	320	14.4	305.6	1,372	717	717	725	176	3,707		1.265	0.046	680	15,740	86.1	14,090
128	64	64	64	320	14.4	305.6	1,372	717	717	725	176	3,707		1.265	0.046	680	15,830	86.1	14,090
128	77	64	64	333	14.9	318.1	1,372	863	717	725	185	3,862	11,260	1.280	0.046	700	16,050	85.8	14,150
128	77	77	64	346	15.5	330.5	1,372	863	863	725	195	4,018		1.293	0.046	730	16,250	85.4	14,240
128	77	77	64	346	15.5	330.5	1,372	863	863	725	195	4,018		1.293	0.046	730	17,290	85.4	14,240
128	110	77	64	379	17.2	361.8	1,372	1,260	863	725	224	4,444	12,030	1.343	0.047	820	18,110	84.3	14,570
128	110	110	64	412	18.6	393.4	1,372	1,260	1,260	725	249	4,866		1.386	0.047	920	18,830	83.3	14,850
128	110	110	64	412	18.6	393.4	1,372	1,260	1,260	725	249	4,866		1.386	0.047	920	19,330	83.3	14,850
145	110	110	64	429	19.4	409.6	1,596	1,260	1,260	725	265	5,106	12,370	1.422	0.047	1,020	19,980	82.7	15,070
160	110	110	64	444	20.1	423.9	1,768	1,260	1,260	725	281	5,294		1.455	0.047	1,100	20,570	82.1	15,210
160	110	110	64	444	20.1	423.9	1,768	1,260	1,260	725	281	5,294		1.455	0.047	1,100	21,500	82.1	15,210
160	110	110	70	450	20.4	429.6	1,768	1,260	1,260	806	288	5,382	12,980	1.472	0.047	1,150	21,830	81.8	15,320
160	110	110	80	460	20.8	439.2	1,768	1,260	1,260	933	300	5,521		1.500	0.047	1,260	22,410	81.4	15,440

TABLE XXV

Turbine-Generator Loadings and Inputs

Turbine-Generator Loads Megawatts								Input to Turbine-Generators Million Btu per Hour								Turbine-Generator Incremental Rate Btu per Kw-hr (R_t)
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	
15	15	15	15	25	25	35	35	226	218	230	228	330	307	428	424	
15	15	15	15	25	25	45	35	226	218	230	228	330	307	522	424	9,385
15	15	15	15	25	25	55	35	226	218	230	228	330	307	616	424	
15	15	15	15	25	25	61	35	226	218	230	228	330	307	672	424	
15	15	15	15	25	25	61	45	226	218	230	228	330	307	672	521	
15	15	15	15	25	25	61	55	226	218	230	228	330	307	672	619	9,727
15	15	15	15	25	25	61	65	226	218	230	228	330	307	672	716	
15	15	15	15	25	25	61	79	226	218	230	228	330	307	672	852	
15	15	15	25	25	25	61	79	226	218	230	330	330	307	672	852	
15	15	15	35	25	25	61	79	226	218	230	432	330	307	672	852	10,222
15	15	15	45	25	25	61	79	226	218	230	535	330	307	672	852	
15	15	15	51	25	25	61	79	226	218	230	596	330	307	672	852	
15	15	15	51	25	25	70	79	226	218	230	596	330	307	764	852	
15	15	15	51	25	25	75	79	226	218	230	596	330	307	816	852	10,263
15	15	15	51	25	25	80	79	226	218	230	596	330	307	867	852	
15	15	15	51	25	25	80	85	226	218	230	596	330	307	867	914	
15	15	15	51	25	25	80	90	226	218	230	596	330	307	867	965	10,294
15	15	15	51	25	25	80	96	226	218	230	596	330	307	867	1027	
15	15	15	51	30	25	80	96	226	218	230	596	382	307	867	1027	
15	15	15	51	35	25	80	96	226	218	230	596	434	307	867	1027	10,400
15	15	15	51	40	25	80	96	226	218	230	596	486	307	867	1027	
15	15	15	51	45	25	80	96	226	218	230	596	538	307	867	1027	
15	15	15	51	45	30	80	96	226	218	230	596	538	360	867	1027	
15	15	15	51	45	35	80	96	226	218	230	596	538	413	867	1027	10,600
15	15	15	51	45	40	80	96	226	218	230	596	538	466	867	1027	
15	15	15	51	45	45	80	96	226	218	230	596	538	519	867	1027	
15	15	15	51	45	45	85	96	226	218	230	596	538	519	920	1027	
15	15	15	51	45	45	90	96	226	218	230	596	538	519	973	1027	10,600
15	15	15	51	45	45	95	96	226	218	230	596	538	519	1026	1027	
15	15	15	51	45	50	95	96	226	218	230	596	538	572	1026	1027	
15	15	15	51	45	55	95	96	226	218	230	596	538	626	1026	1027	10,667
15	15	15	51	45	60	95	96	226	218	230	596	538	679	1026	1027	
15	15	15	51	45	66	95	96	226	218	230	596	538	743	1026	1027	
15	15	15	51	45	66	105	96	226	218	230	596	538	743	1134	1027	
15	15	15	51	45	66	115	96	226	218	230	596	538	743	1242	1027	10,793
15	15	15	51	45	66	124	96	226	218	230	596	538	743	1339	1027	
15	15	15	51	45	66	124	105	226	218	230	596	538	743	1339	1124	
15	15	15	51	45	66	124	115	226	218	230	596	538	743	1339	1232	10,793
15	15	15	51	45	66	124	125	226	218	230	596	538	743	1339	1340	
15	15	15	51	50	66	124	125	226	218	230	596	592	743	1339	1340	
15	15	15	51	55	66	124	125	226	218	230	596	646	743	1339	1340	10,818
15	15	15	51	60	66	124	125	226	218	230	596	700	743	1339	1340	
15	15	15	51	67	66	124	125	226	218	230	596	776	743	1339	1340	
15	15	20	51	67	66	124	125	226	218	284	596	776	743	1339	1340	
15	15	25	51	67	66	124	125	226	218	338	596	776	743	1339	1340	10,842
15	15	30	51	67	66	124	125	226	218	393	596	776	743	1339	1340	
15	15	34	51	67	66	124	125	226	218	436	596	776	743	1339	1340	

TABLE XXV—*Continued*

Turbine-Generator Loadings and Inputs

Turbine-Generator Loads Megawatts								Input to Turbine-Generators Million Btu per Hour								Turbine-Generator Incremental Rate Btu per Kw-hr (R_t)
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	
20	15	34	51	67	66	124	125	280	218	436	596	776	743	1339	1340	10,857
25	15	34	51	67	66	124	125	335	218	436	596	776	743	1339	1340	
30	15	34	51	67	66	124	125	389	218	436	596	776	743	1339	1340	
36	15	34	51	67	66	124	125	454	218	436	596	776	743	1339	1340	
36	15	34	51	72	66	124	125	454	218	436	596	830	743	1339	1340	10,875
36	15	34	51	77	66	124	125	454	218	436	596	885	743	1339	1340	
36	15	34	51	83	66	124	125	454	218	436	596	950	743	1339	1340	
36	15	36	51	83	66	124	125	454	218	458	596	950	743	1339	1340	10,875
36	15	38	51	83	66	124	125	454	218	480	596	950	743	1339	1340	
36	15	41	51	83	66	124	125	454	218	512	596	950	743	1339	1340	
36	15	41	55	83	66	124	125	454	218	512	640	950	743	1339	1340	11,000
36	15	41	60	83	66	124	125	454	218	512	695	950	743	1339	1340	
36	15	41	65	83	66	124	125	454	218	512	750	950	743	1339	1340	
36	15	41	70	83	66	124	125	454	218	512	805	950	743	1339	1340	
36	15	41	70	83	70	124	125	454	218	512	805	950	788	1339	1340	11,125
36	15	41	70	83	75	124	125	454	218	512	805	950	843	1339	1340	
36	15	41	70	83	82	124	125	454	218	512	805	950	921	1339	1340	
36	15	41	70	83	82	124	135	454	218	512	805	950	921	1339	1453	11,286
36	15	41	70	83	82	124	145	454	218	512	805	950	921	1339	1566	
36	15	41	70	83	82	124	160	454	218	512	805	950	921	1339	1735	
36	15	41	70	90	82	124	160	454	218	512	805	1030	921	1339	1735	11,481
36	15	41	70	100	82	124	160	454	218	512	805	1145	921	1339	1735	
36	15	41	70	110	82	124	160	454	218	512	805	1260	921	1339	1735	
36	15	41	70	110	90	124	160	454	218	512	805	1260	1013	1339	1735	11,500
36	15	41	70	110	100	124	160	454	218	512	805	1260	1128	1339	1735	
36	15	41	70	110	110	124	160	454	218	512	805	1260	1243	1339	1735	
36	20	41	70	110	110	124	160	454	276	512	805	1260	1243	1339	1735	11,600
36	25	41	70	110	110	124	160	454	334	512	805	1260	1243	1339	1735	
36	30	41	70	110	110	124	160	454	392	512	805	1260	1243	1339	1735	
36	35	41	70	110	110	124	160	454	450	512	805	1260	1243	1339	1735	
36	35	41	70	110	110	135	160	454	450	512	805	1260	1243	1467	1735	11,667
36	35	41	70	110	110	145	160	454	450	512	805	1260	1243	1584	1735	
36	35	41	70	110	110	160	160	454	450	512	805	1260	1243	1759	1735	
36	37	41	70	110	110	160	160	454	475	512	805	1260	1243	1759	1735	12,571
36	39	41	70	110	110	160	160	454	500	512	805	1260	1243	1759	1735	
36	42	41	70	110	110	160	160	454	538	512	805	1260	1243	1759	1735	
36	42	41	73	110	110	160	160	454	538	512	845	1260	1243	1759	1735	13,400
36	42	41	76	110	110	160	160	454	538	512	885	1260	1243	1759	1735	
36	42	41	80	110	110	160	160	454	538	512	939	1260	1243	1759	1735	
40	42	41	80	110	110	160	160	511	538	512	939	1260	1243	1759	1735	14,214
45	42	41	80	110	110	160	160	582	538	512	939	1260	1243	1759	1735	
50	42	41	80	110	110	160	160	653	538	512	939	1260	1243	1759	1735	
50	45	41	80	110	110	160	160	653	581	512	939	1260	1243	1759	1735	14,250
50	48	41	80	110	110	160	160	653	624	512	939	1260	1243	1759	1735	
50	50	41	80	110	110	160	160	653	652	512	939	1260	1243	1759	1735	
50	50	43	80	110	110	160	160	653	652	543	939	1260	1243	1759	1735	15,333
50	50	46	80	110	110	160	160	653	652	589	939	1260	1243	1759	1735	
50	50	50	80	110	110	160	160	653	652	650	939	1260	1243	1759	1735	

it will usually be found that single curves for the auxiliary power consumptions are satisfactory. Where refinement is desired, curves of auxiliary power consumption can be established for the different combinations of turbines and boilers. It is to be noted that in order to apply equation 63 it is necessary to know the boiler output corresponding to the station load, since the boiler incremental rate R_b is plotted as a function of the boiler output. For the determination of the boiler output it will be found convenient to tabulate the turbine-generator loads and corresponding inputs under incremental loading on the assumption that all the units are in operation on the bus. This is illustrated in Table XXV for a station with eight turbine-generator units. From this table, the loading, the input to the turbine room, and the value of R_t can readily be obtained for any combination of units in operation by omitting the tabulated values for those units not in the combination. For example, referring to Table XXV, if units 1, 3, 5, and 7 are in service, then for a total load of 281 megawatts the following would obtain:

Unit	Load, megawatts	Input, 10^6 Btu per hour
1	36	454
3	38	480
5	83	950
7	124	1,339
Total	281	3,223

The boiler-room output is obtained by adding the input to the steam-driven auxiliaries to the turbine-room input. The value of R_t is indicated in Table XXV as 10,875 Btu per kw-hr. The values of R_e, R_b, and R_s are obtained from the appropriate curves so that the station net incremental heat rate can be obtained by substituting the values in equation 63.

Effect of Steam Tie Line of Limited Capacity

In developing equations 47, 56, and 63 it was assumed that any boiler could supply steam to the throttle of any turbine without limitation in respect to quantity and direction of flow. When all the boilers and turbines are connected to a common steam header, the quantity of steam that can be transferred between adjacent sections of the steam header is sometimes limited. Considering this limitation with reference to two boilers and two turbines, it may no longer be possible to assume that L_1 and O_1 are independent variables. Assume, therefore, that the maximum heat that can be transferred through the steam tie line is H_0. As long as the difference between O_1 and T_1 is less than H_0, the turbines and

boilers can be loaded independently. When the transfer through the tie reaches H_0, equations 47 and 53 no longer hold. There is only one independent variable, say L_1, and equation 44 becomes

$$\frac{dI_t}{dL_1} = 0 \qquad [64]$$

$$= \frac{d(I_1 + I_2)}{dL_1} = \frac{dI_1}{dL_1} + \left(\frac{dI_2}{dL_2} \times \frac{dL_2}{dL_1}\right)$$

$$= \frac{dI_1}{dL_1} - \frac{dI_2}{dL_2}$$

or

$$\frac{dI_1}{dL_1} = \frac{dI_2}{dL_2} \qquad [65]$$

and

$$\frac{dI_1}{dO_1} \times \frac{dO_1}{dL_1} = \frac{dI_2}{dO_2} \times \frac{dO_2}{dL_2} \qquad [66]$$

But since

$$O_1 = T_1 + H_0$$

and

$$O_2 = T_2 - H_0$$

H_0 being a constant, it follows that

$$\frac{dO_1}{dL_1} = \frac{dT_1}{dL_1}$$

and

$$\frac{dO_2}{dL_2} = \frac{dT_2}{dL_2}$$

hence

$$\frac{dI_1}{dO_1} \times \frac{dT_1}{dL_1} = \frac{dI_2}{dO_2} \times \frac{dT_2}{dL_2} \qquad [67]$$

Equation 67 simply implies that, whenever independent loading of the turbines and boilers would require the transfer of a quantity of steam in excess of the steam tie capacity, the total station load should be incrementally divided between the sections. The incremental rate for each section is established with capacity load on the steam tie, so that dI_1/dO_1 is the incremental rate of boiler 1 corresponding to an output of $T_1 + H_0$ and dI_2/dO_2 is the incremental rate of boiler 2 corresponding to an output of $T_2 - H_0$. At these boiler outputs, there is a transfer of steam equal to the capacity of the steam tie.

The steps involved in the procedure for determining the proper load division are illustrated by Fig. 61 and Table XXVI. Consider the hypothetical set-up shown in Fig. 61*A*, consisting of two turbines, each supplied by a group of boilers which are in parallel through a tie line of limited capacity. The performance curves for the individual turbines and the corresponding groups of boilers are shown in Figs. 61*B* and 61*C*,

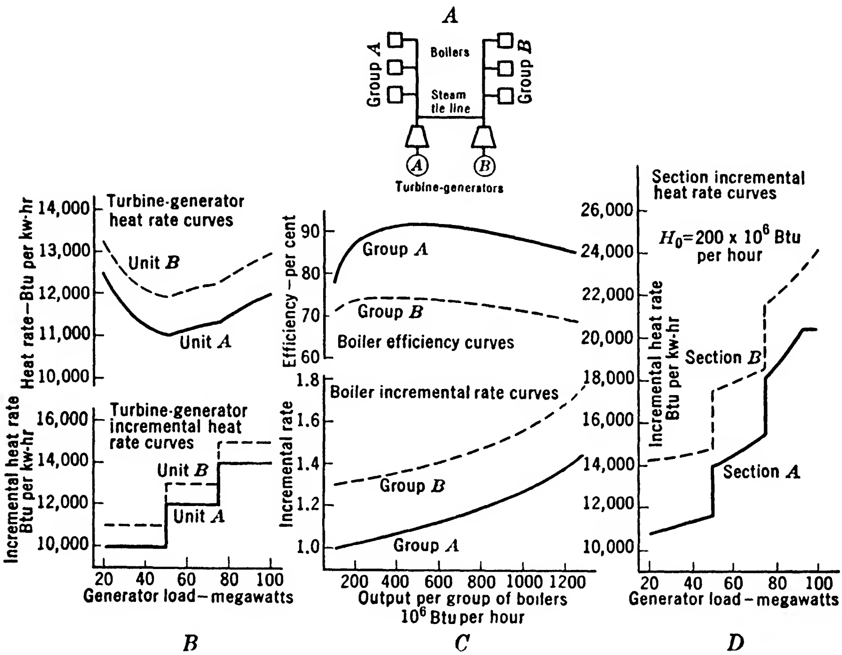


FIG. 61. Effect of steam tie line of limited capacity on incremental loading.

respectively. The turbine-generators were assumed to have a rated capacity of 100 megawatts each, with minimum operating loads of 20 megawatts. Each group of boilers was assumed to have a capacity of 1,300 million Btu per hour with a minimum operating output of 100 million Btu per hour. The team tie line was assumed to have a limited capacity equivalent to 200 million Btu per hour.

Since the capacity of the steam tie line may be a limitation to the most economical load division, it is necessary to determine in what load range, if any, this limitation will occur. Step 1 of Table XXVI indicates the load division when the turbines and boilers are loaded incrementally and independently of each other using the respective incremental rate curves of Figs. 61*B* and 61*C*. It is seen from step 1 that independent loading cannot be applied for station loads in the range from 80 to 155

TABLE XXVI

EFFECT OF STEAM TIE LINE LIMITATION ON LOAD DIVISION

Step 1

INDEPENDENT LOADING OF TURBINES AND BOILERS

Generator Loads Megawatts			Turbine Inputs 10^6 Btu per Hour			Boiler Outputs 10^6 Btu per Hour		Steam Tie Line Load 10^6 Btu per hour
Total L	Unit A L_a	Unit B L_b	Unit A T_a	Unit B T_b	Total O_t	Group A O_a	Group B O_b	H
40	20	20	250	265	515	415	100	165
50	30	20	350	265	615	515	100	165
60	40	20	450	265	715	615	100	165
70	50	20	550	265	815	715	100	165
80	50	30	550	375	925	825	100	275
90	50	40	550	485	1,035	935	100	385
100	50	50	550	595	1,145	1,045	100	495
110	60	50	670	595	1,265	1,085	180	415
120	70	50	790	595	1,385	1,115	270	325
125	75	50	850	595	1,445	1,120	325	270
135	75	60	850	725	1,575	1,165	410	315
145	75	70	850	855	1,705	1,195	510	345
150	75	75	850	920	1,770	1,210	560	360
155	80	75	920	920	1,840	1,230	610	310
165	90	75	1,060	920	1,980	1,260	720	200
175	100	75	1,200	920	2,120	1,300	820	100
180	100	80	1,200	995	2,195	1,300	895	100
190	100	90	1,200	1,145	2,345	1,300	1,045	100
200	100	100	1,200	1,295	2,495	1,300	1,195	100

TABLE XXVI—*Continued*

EFFECT OF STEAM TIE LINE LIMITATION ON LOAD DIVISION

Step 2

COMPUTATION OF ADJUSTED SECTION INCREMENTAL RATES

	Load	Turbine Incremental Rate R_t	Turbine Input T	Capacity of Steam Tie Line H_o	Boiler Output $(T \pm H_o)$ O	Boiler Incremental Rate R_b	Section Incremental Rate $(R_t \times R_b)$ R_{st}
Section A	20	10,000	250	200	450	1.080	10,800
	30		350		550	1.108	11,080
	40		450		650	1.136	11,360
	50		550		750	1.170	11,700
	50	12,000	550		750	1.170	14,040
	55		610		810	1.190	14,280
	60		670		870	1.210	14,520
	65		730		930	1.238	14,860
	70		790		990	1.265	15,180
	75		850		1,050	1.295	15,540
	75	14,000	850		1,050	1.295	18,130
	80		920		1,120	1.332	18,650
	85		990		1,190	1.378	19,290
	90		1,060		1,260	1.427	19,980
	92.9		1,100		1,300	1.460	20,440
	95		1,130		1,300	1.460	20,440
	100		1,200		1,300	1.460	20,440
Section B	20	11,000	265	200	100	1.300	14,300
	23.2		300		100	1.300	14,300
	30		375		175	1.310	14,410
	40		485		285	1.330	14,630
	50		595		395	1.354	14,890
	50	13,000	595		395	1.354	17,600
	55		660		460	1.370	17,810
	60		725		525	1.385	18,010
	65		790		590	1.400	18,200
	70		855		655	1.420	18,460
	75		920		720	1.440	18,720
	75	15,000	920		720	1.440	21,600
	80		995		795	1.465	21,980
	85		1,070		870	1.494	22,410
	90		1,145		945	1.527	22,910
	95		1,220		1,020	1.567	23,510
	100		1,295		1,095	1.610	24,150

TABLE XXVI—*Continued*

EFFECT OF STEAM TIE LINE LIMITATION ON LOAD DIVISION

Step 3

LOADING ADJUSTED TO STEAM TIE LINE CAPACITY

Generator Loads			Turbine Inputs			Boiler Outputs		Steam Tie Line Load H_o
Total L	Unit A L_a	Unit B L_b	Unit A T_a	Unit B T_b	Total O_t	Group A O_a	Group B O_b	
80	56	24	622	309	931	822	109	200
90	58.5	31.5	652	392	1,044	852	192	200
100	61	39	682	474	1,156	882	274	200
110	64	46	718	551	1,269	918	351	200
120	70	50	790	595	1,385	990	395	200
125	75	50	850	595	1,445	1,050	395	200
135	75	60	850	725	1,575	1,050	525	200
145	77	68	878	829	1,707	1,078	629	200
150	78.5	71 5	899	875	1,774	1,099	675	200
155	85	75	990	920	1,910	1,190	720	200

megawatts, as this would require a transfer of steam between the sections in excess of the rated capacity of the steam tie. For this range of station load it is necessary to divide the load incrementally between sections on the basis defined by equation 67. The computation of the section incremental rates adjusted for capacity flow through the steam tie line is shown as step 2 of Table XXVI, and the respective incremental rate curves for the sections are shown in Fig. 61*D*. Using these curves, the proper load division between the boilers and turbines for station loads between 80 and 155 megawatts is shown as step 3 of Table XXVI.

It is now possible to formulate some general principles and rules as a guide for the application of incremental loading within the generating station. These may be briefly summarized as follows:

1. If the design of the station piping is such that all boilers feed into a common header, permitting any boiler to supply steam to any turbine, then the maximum station efficiency will be obtained when the boilers and turbines are independently loaded, so that all the turbines are operating at loads which correspond to the same turbine incremental rate, and, similarly, all the boilers are operating at outputs which correspond to the same boiler incremental rate.

2. If certain groups of turbines can be supplied with steam only by certain boilers, then the independent treatment of the boiler and turbine rooms is not possible. Under these conditions it is necessary to compute the combined performance of each section of the station, consisting of a group of turbines and the particular group of boilers which supply them with steam. Each section is treated as an independent station, and the turbines and boilers of that section are loaded independently. The overall station performance is obtained by dividing the total station load incrementally between sections.

3. If sections of a station are connected by a steam tie line of limited capacity, the boiler and turbine rooms may be treated independently until the limit of the tie-line capacity is reached. Beyond this point the total station load is divided between the sections incrementally, the incremental rates of each section being calculated for the condition of capacity flow through the steam tie line.

4. Where two sections of a station operating at different steam pressures are in parallel through a pressure-reducing valve and perhaps through a steam desuperheater, then, up to the capacity of the reducing valve or desuperheater, the turbine and boiler rooms may be independently treated in the manner discussed above, with the additional limitation that the flow of steam between sections must be in one direction only. If independent loading results in steam flow through the steam tie in the wrong direction, the station load should be divided incrementally

between the sections as described in 2 above. The limitation in regard to direction means that H_0 cannot be negative in the derivation of equation 67 and must have a value between zero and H_0. If independent loading results in steam flow in the right direction but in amounts exceeding the capacity of the reducing valve or desuperheater, the station load should be incrementally divided between the sections in the manner described in 3 above.

The operation of a station at two different pressures, with a steam tie between the sections through a pressure-reducing valve, may introduce difficulties with respect to proper load division due to the complicated arrangements that may be necessary to return the feedwater from the low- to the high-pressure section. The procedure then would depend upon the piping arrangement of the station.

Checking Station Performance

After the station performance curves have been established it is desirable to check them against the actual performance of the station for some specified period. Since it is the customary practice to report station performance for monthly periods, the check should be made for several monthly periods, preferably for a month during each of the seasons: winter, summer, spring, and fall. Essentially, the procedure is to compute the station heat rate by applying the computed heat rate curves to the actual load cycles. The integrated results will indicate the effect of the load cycle on the station heat rate, but it will not include those station losses which are independent of the load cycle and which cannot be incorporated in the computed heat rate and incremental heat rate curves. It is necessary therefore to make the computations in two steps. In the first step, the station "load heat rate" is computed. The load heat rate is considered to be a direct function of the instantaneous load on the station. In the second step, all the station losses which are independent of the instantaneous loads on the station are computed. This will be referred to as the station "loss heat rate." The sum of the two gives the overall heat rate, which is then compared with the officially reported heat rate for the station.

Load Heat Rate

To calculate the load heat rate it is necessary to analyze the actual operating conditions which prevailed in the station during the period, and the curves which were computed for these conditions should be applied. The important items which must be established are:

1. A load duration curve for each combination of turbine-generators actually operated.

TABLE XXVII

Turbine-Generator Combinations and Hours Duration

Date October	2 Units					3 Units						4 Units				5 Units	Total Hours
	Units No.					Units No.						Units No.				Units No.	
	7-8	6-8	6-7	5-7	5-8	6-7-8	5-6-7	5-6-8	5-7-8	4-7-8	4-6-7	5-6-7-8	4-6-7-8	1-6-7-8	2-5-7-8	4-5-6-7-8	
1	0.5	15				8.5											24
2	9					9.5				0.5			5				24
3	9					9.5				0.5			5				24
4	8					9.5					1		5.5				24
5			6.5			2	1.5				0.5	7	4			2.5	24
6			6			10	1	1.5				5.5					24
7					7.5	2.5		0.5	9			4.5					24
8	16	0.5				7.5											24
9	0.5	7				11.5						5					24
10	8					8.5	1					6.5					24
11				6.5			1.5		9.5			6.5					24
12			6			8	1.5	1.5				6	1				24
13		6.5				2		2				13.5					24
14		7.5				10.5		0.5				5		0.5			24
15	17.5					6.5											24
16	8.5					10.5						5					24
17	8.5					3						12.5					24
18	8						1.5		7.5			7					24
19				6.5			1	1.5	9			6					24
20					6			2.5	2			13.5					24
21					6.5	1.5		1	8.5			6			0.5		24
22	16.5								7.5								24
23	0.5			6.5		1.5			2.5			13					24
24	8.5					4						11.5					24
25	7.5					9	1.5					6					24
26			7			9		2				6					24
27	1	7				2.5			1.5			12					24
28	13							1	3.5			6.5					24
29	1	7.5				6.5		9									24
30	8					8.5	1.5					6					24
31	1		6.5			10.5	1					5					24
Monthly Totals	150.5	51	32	19.5	20	172.5	13	23	60.5	1	1.5	175.5	20.5	0.5	0.5	2.5	744

TABLE XXVIII

Load Duration Data for Units 6-7-8

Date October	Load, Megawatts 130	135	140	145	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250	255	260	265	270	275	280	285	290	295	300	305	310	Daily Total
1			1		0.5		0.5				1						0.5						0.5		0.5		0.5		1.5		1.5		0.5					8.5
2																			0.5				0.5				1.5		2.5		4.5							9.5
3																			0.5								1			0.5	1.5		6					9.5
4																						0.5					0.5		0.5	1.5	1	2.5	2	0.5	0.5			9.5
5			0.5														0.5				0.5																0.5	2
6					0.5								0.5						0.5						0.5		2		1		5							10
7													0.5									0.5					0.5				0.5		0.5					2.5
8					0.5				0.5		0.5						1				0.5				0.5					1	3							7.5
9					0.5						0.5		0.5						1						1.5				1.5		5.5		0.5					11.5
10																							0.5				0.5		0.5		3.5		3.5					8.5
11																																						
12			0.5										0.5								1		2	0.5	3.5													8
13															0.5								0.5								0.5				0.5			2
14							0.5								1	0.5	2	0.5	0.5		3		1			0.5	0.5		0.5									10.5
15					0.5										1						0.5						1		1		2.5							6.5
16																	0.5				0.5		0.5		0.5		0.5		1.5		4.5	0.5	1		0.5			10.5
17															0.5								1						0.5		0.5		0.5					3
18																																						
19																																						
20																																						
21																	0.5				0.5								0.5									1.5
22																																						
23															0.5						0.5								0.5									1.5
24													0.5		0.5										1				0.5				0.5	1				4
25											0.5		0.5						0.5				0.5				2.5		4.5									9
26	0.5								0.5						0.5						0.5		0.5				5		1.5									9
27	0.5										0.5						0.5						0.5						0.5									2.5
28																																						
29											0.5		0.5						0.5				0.5				0.5		1.5		2.5							6.5
30																	0.5				0.5		0.5		2.5		3		1		0.5							8.5
31											0.5		0.5		0.5						1.5		0.5		0.5		4		2.5									10.5
Monthly Total	1		2		2.5		1		1		4		4		5	0.5	6	0.5	4		9.5	1	9.5	0.5	11	0.5	23.5		24	3	37	3	15	1.5	1.5		0.5	172.5

Duration in hours.

TABLE XXIX

Calculation of Station Heat Input for Units 6-7-8

Load, Megawatts			Duration Hours	Generation, Megawatt Hours		Station Net Heat Rate Btu per Kw-hr	Station Heat Input Million Btu
Gross	Electric Auxiliaries	Net		Gross	Net		
130	6.5	123.5	1.0	130.0	123.5	14,060	1,736.4
140	6.8	133.2	2.0	280.0	266.4	13,800	3,676.3
150	7.2	142.8	2.5	375.0	357.0	13,620	4,862.3
160	7.6	152.4	1.0	160.0	152.4	13,490	2,055.9
170	8.0	162.0	1.0	170.0	162.0	13,360	2,164.3
180	8.4	171.6	4.0	720.0	686.4	13,330	9,149.7
190	8.8	181.2	4.0	760.0	724.8	13,260	9,610.9
200	9.2	190.8	5.0	1,000.0	954.0	13,240	12,631.0
205	9.4	195.6	0.5	102.5	97.8	13,230	1,293.9
210	9.6	200.4	6.0	1,260.0	1,202.4	13,220	15,895.7
215	9.8	205.2	0.5	107.5	102.6	13,220	1,356.4
220	10.0	210.0	4.0	880.0	840.0	13,210	11,096.4
230	10.4	219.6	9.5	2,185.0	2,086.2	13,200	27,537.8
235	10.6	224.4	1.0	235.0	224.4	13,190	2,959.8
240	10.8	229.2	9.5	2,280.0	2,177.4	13,180	28,698.1
245	11.0	234.0	0.5	122.5	117.0	13,200	1,544.4
250	11.3	238.7	11.0	2,750.0	2,625.7	13,210	34,685.5
255	11.5	243.5	0.5	127.5	121.8	13,210	1,609.0
260	11.7	248.3	23.5	6,110.0	5,835.1	13,220	27,140.0
270	12.1	257.9	24.0	6,480.0	6,189.6	13,260	82,074.1
275	12.4	262.6	3.0	825.0	787.8	13,280	10,462.0
280	12.6	267.4	37.0	10,360.0	9,893.8	13,300	131,587.5
285	12.8	272.2	3.0	855.0	816.6	13,320	10,877.1
290	13.0	277.0	15.0	4,350.0	4,155.0	13,350	55,469.3
295	13.2	281.8	1.5	442.5	422.7	13,380	5,655.7
300	13.4	286.6	1.5	450.0	429.9	13,400	5,760.7
310	13.9	296.1	0.5	155.0	148.1	13,440	1,990.5
Totals			172.5	43,672.5	41,700.4	13,275*	553,580.7

* Weighted average.

TABLE XXX

CALCULATION OF STATION LOAD HEAT RATE

CIRCULATING WATER INLET TEMPERATURE = 63° F

SUMMARY SHEET

Number of Units	Turbine-Generator Combination	Duration Hours	Generation, Mw-Hr Gross	Generation, Mw-Hr Net	Station Heat Input Million Btu	Station Load Heat Rate Btu per Net Kw-hr
2	7–8	150.5	18,517.5	17,579.4	238,677.8	13,577
	6–8	51.5	5,262.5	4,981.8	70,486.0	14,149
	6–7	32.0	3,345.0	3,167.7	44,749.1	14,127
	5–7	19.5	2,155.0	2,042.7	28,945.6	14,170
	5–8	20.0	2,120.0	2,008.1	28,696.4	14,290
	Total	273.0	31,400.0	29,779.7	411,554.9	13,820 *
3	6–7–8	172.5	43,672.5	41,700.4	553,580.7	13,275
	5–6–7	13.0	2,190.0	2,086.5	28,781.1	13,794
	5–6–8	23.0	3,720.0	3,543.0	49,161.2	13,876
	5–7–8	60.5	14,480.0	13,919.2	187,297.9	13,456
	4–6–7	1.5	245.0	233.4	3,173.3	13,596
	4–7–8	1.0	220.0	210.0	2,778.6	13,232
	Total	271.5	64,527.5	61,692.5	824,772.8	13,369 *
4	5–6–7–8	175.5	57,427.5	54,845.5	757,360.1	13,809
	4–6–7–8	20.5	6,720.0	6,417.6	88,115.6	13,730
	1–6–7–8	0.5	122.5	117.0	1,581.6	13,518
	2–5–7–8	0.5	110.0	105.0	1,432.8	13,646
	Total	197.0	64,380.0	61,485.1	848,490.1	13,800 *
5	4–5–6–7–8	2.5	865.0	825.9	11,569.8	14,009
	Grand Total	744.0	161,172.5	153,783.2	2,096,387.6	13,632 *

* Weighted average.

2. The average circulating water inlet temperature for the period.

3. The combination of boilers in service. If several combinations of boilers were operated, then item 1 should be established for each combination of boilers.

4. The gross and net outputs for the period.

To illustrate the procedure involved, an actual calculation will be described for a station consisting of 8 turbine-generators and 32 boilers. To simplify the discussion a month has been selected during which only one combination of boilers was operated.

From the station log sheets, a tabulation similar to Table XXVII is compiled. This indicates the various turbine-generator combinations and the number of hours of actual operation for each. The sum of these hours should equal the total period hours. As a cross-check, the operating hours for each combination should add to the total daily hours of 24 as shown in the last column.

The load duration data for each combination of units should next be determined from the daily log sheets. A sample calculation is shown in Table XXVIII for a three-unit combination. The daily and monthly durations should check with the corresponding values of Table XXVII. For example, referring to Table XXVIII it will be seen that, for October 16, the daily total duration hours indicated in the last column amount to 10.5, while the monthly total for this combination is 172.5. The same values are indicated in the seventh column of Table XXVII; thus this table serves as a means of checking against errors.

Upon completion of the load duration data for each combination of units, the corresponding station heat inputs are calculated as shown in Table XXIX. The gross loads and corresponding hourly durations are obtained from Table XXVIII. The electric auxiliary power consumption and the station net heat rates are obtained from the computed curves for the station.

The summary for all combinations of units, shown in Table XXX, is self-explanatory. The station load heat rate computed for the month in question is indicated as 13,632 Btu per net kw-hr.

Loss Heat Rate

Table XXXI shows the station loss heat rate. In this particular station the fixed losses are comparatively small, being about 1 per cent of the overall station heat rate. Hence, a comprehensive analysis beyond that indicated in the table cannot be justified. The loss heat rate for the station is indicated in the table as 152 Btu per net kw-hr.

The computed overall station heat rate is the sum of the load heat

TABLE XXXI

Calculation of Station Loss Heat Rate

Net Generation (Computed)				153,783.2 mw-hr
Loss Due to Turbine-Generator Starts				
Units 1, 2, 3 & 4	6 starts	@ 60,000 lb steam	= 360,000 lb	
5, 6	39	@ 85,000	= 3,315,000	
7, 8	17	@ 150,000	= 2,555,000	
		Total	6,230,000 lb	
Enthalpy above make-up water enthalpy			1,330 Btu per lb	
Average boiler-room efficiency			85.7%	
Loss heat rate		$\frac{6{,}230{,}000 \times 1{,}330}{153{,}783{,}200 \times 0.857} =$		62 Btu per net kw-hr
Loss Due to Cold Boiler Starts				
8 starts @ 25,000 lb Coal			= 200,000 lb	
4 @ 40,000			= 160,000	
Total			360,000 lb	
Calorific value of coal per pound			14,020 Btu	
Loss heat rate		$\frac{360{,}000 \times 14{,}020}{153{,}783{,}200} =$		33
Loss Due to Blowdown				
Total blowdown		11,289,000 lb		
Enthalpy per pound		400 Btu		
Loss heat rate		$\frac{11{,}289{,}000 \times 400}{153{,}783{,}200 \times 0.857} =$		34
Loss Due to Station Heating				
Total heating steam		2,600,000 lb		
Enthalpy per pound		1,174 Btu		
Loss heat rate		$\frac{2{,}600{,}000 \times 1{,}174}{153{,}783{,}200 \times 0.857} =$		23
Station Loss Heat Rate				152 Btu per net kw-hr

rate and loss heat rate which in this case amounts to 13,632 + 152 or 13,784 Btu per net kw-hr.

The loss heat rate is a function of the station losses, which may represent from 1 to 10 per cent of the overall station heat rate, depending upon the design of and the operating conditions within the station. When the losses are relatively small, there is justification for only the simplest kind of analysis, since a relatively large error in the calculated loss heat rate affects the calculated overall station heat rate but slightly. Thus, for the illustration above, it is seen that the loss heat rate is approximately

TABLE XXXII

SUMMARY OF CALCULATIONS TO CHECK STATION PERFORMANCE

		Btu per net kw-hr
Computed Station Load Heat Rate		13,900
Computed Loss Heat Rates		
	Btu per net kw-hr	
1. Loss due to boiler blowdown	50	
2. Input to coal feed steam injectors	50	
3. Loss due to starting steam auxiliaries, free blows, soot blowing, steam leaks, etc.	285	
4. Loss due to feedwater thrown overboard	45	
5. Loss due to cold boiler starts	140	
6. Loss due to station heating	140	
7. Loss due to turbine-generator starts	80	
8. Loss due to piping radiation	60	
9. Coal lost in mill washers	100	
10. Loss due to reduced steam temperature from standard value	60	
Total station loss heat rate		1,010
Computed overall station heat rate		14,910
Actual heat rate		15,210
Ratio—actual to computed station heat rate		1.020
Actual load heat rate (15,210 − 1,010)		14,200
Ratio—actual to computed load heat rate		1.022

1.1 per cent of the overall station heat rate. An error in the calculated loss heat rate of as much as 20 per cent would introduce an error of only 0.22 per cent in the calculated overall station heat rate.

A more extensive analysis than that shown in Table XXXI is justified if the station losses amount to 5 per cent or more of the overall station heat rate. Table XXXII illustrates the items considered and the results obtained for a station whose losses are approximately 6.6 per cent of the overall station heat rate.

Adjustment of Performance Curves

The purpose of checking computed performance against actual performance is to give assurance that the computed heat rate and incremental heat rate curves correctly define the operating characteristics of the station. In general, it should be possible to calculate the overall station heat rates within ± 2 per cent of actual values. Results obtained for a particular station are shown in Fig. 62 by months for the calendar year

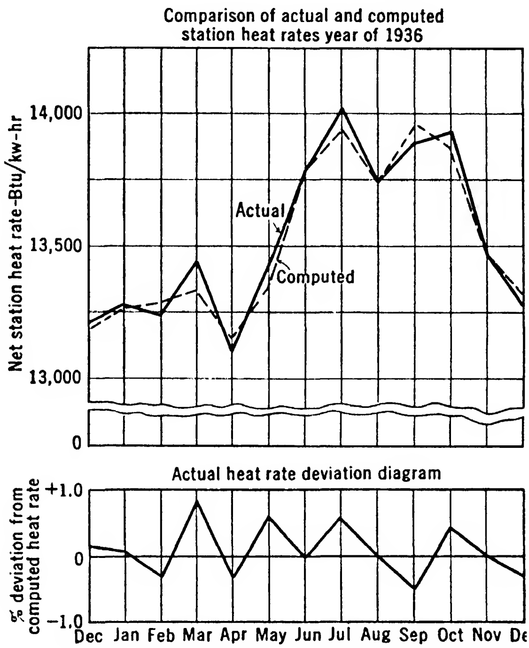


FIG. 62. Comparison of actual and computed monthly heat rates for a generating station.

1936. For this station the indicated variation between the computed and actual station heat rates lies between ± 1 per cent. To obtain such a close check requires extensive analysis of those factors which influence the monthly station heat rates. The extent to which this can be done is illustrated for the same station in Fig. 63, which shows the effect of changes in operating conditions on the station losses expressed in terms of the station heat rate. The indicated losses are measured from "bogey" conditions as datum. They also serve as a means of accounting for the changes in the monthly station heat rate values.

It is sometimes helpful to make separate performance checks for the boiler and turbine rooms, when the computed performance of the station does not check with the actual performance within acceptable limits,

since this may disclose whether the computed performance curves for the boiler room, turbine room, or both need revision. The results of such a

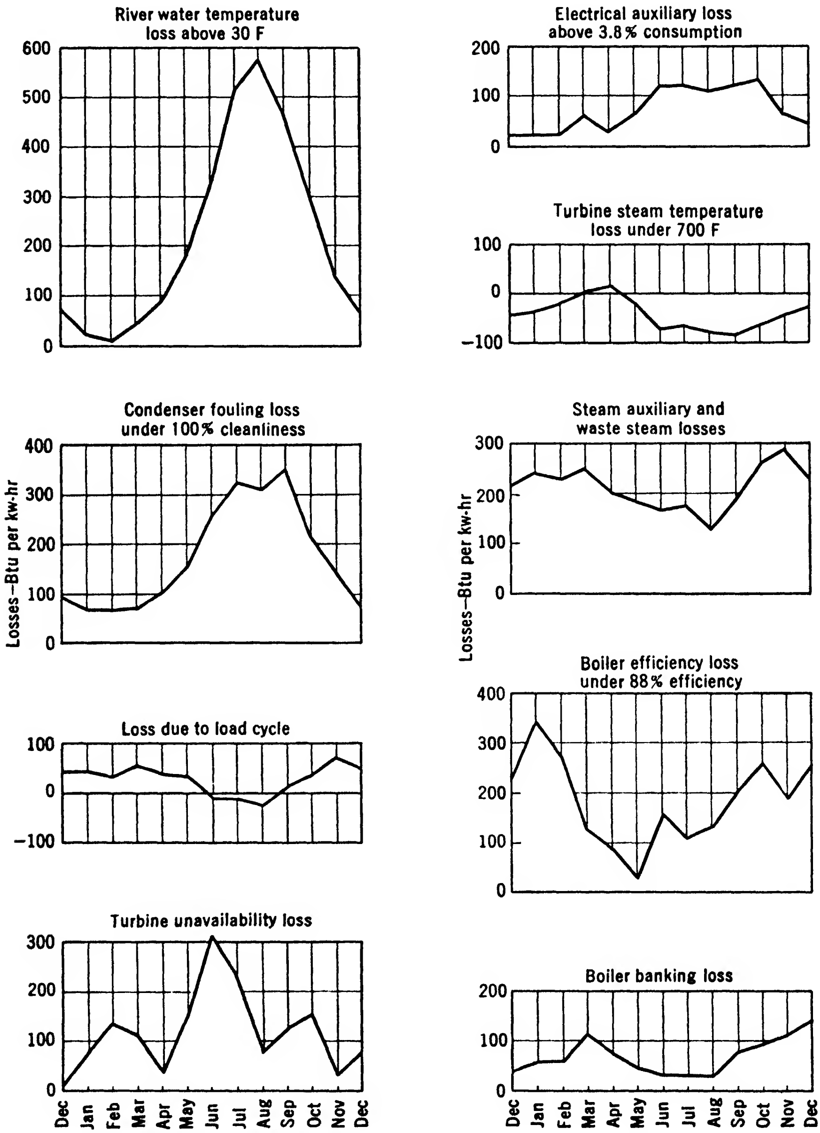


FIG. 63. Effect of changes in operating conditions on generating station losses.

breakdown are shown in Table XXXIII for the station in question. Referring to this table it is seen that the computed performances of the boiler and turbine rooms are in very close agreement with actual operation.

The variation between the computed and actual generation is explained by the fact that the computed value is derived from instantaneous loads recorded on log sheets every half hour. Variation of the load during the half-hour intervals will introduce an error, the magnitude of which will depend upon the degree of variation. Hence it is to be expected that the computed generation will differ from the actual generation, which is usually measured by watt-hour meters. In general, the variation will not exceed 3 per cent and will not materially affect the computed overall station heat rate.

TABLE XXXIII

COMPARISON BETWEEN ACTUAL AND COMPUTED STATION PERFORMANCE

		Actual Performance	Computed Performance	Per Cent Difference
Gross generation	kw-hr	165,531,741	161,172,500	−2.6
Station auxiliaries	kw-hr	6,970,531	7,389,300	+6.0
Net generation	kw-hr	158,561,210	153,783,200	−3.0
Auxiliaries—per cent of gross generation		4.21	4.58	+8.8
Turbine-room overall heat rate	Btu per kw-hr	11,250	11,270	+0.2
Boiler-room overall efficiency	per cent	85.3	85.7	+0.5
Station net heat rate	Btu per kw-hr	13,770	13,780	+0.1

Although the variation in the auxiliary power consumption shown in the table is relatively large, the effect on the computed overall station heat rate is not material. Thus, with auxiliary consumption at the actual value of 4.21 per cent of gross generation, the computed station net heat rate would be 13,730 Btu per kw-hr, instead of the value shown, which would differ from the actual value by only −0.3 per cent and still be well within the desired accuracy.

If the difference between the computed and actual performance of the station is large enough to require adjustment of the computed performance curves, the method of adjustment should be given careful consideration. It is preferable that adjustment of the computed performance curves be avoided whenever it is possible to do so, because there is no assurance that the adjusted curves, regardless of the method used, will define the operating characteristics of the station correctly. Experience has shown repeatedly that adjustments are unnecessary when the basic data are reasonably accurate. For this reason analysis of the basic data for detection of errors and inconsistencies is preferable to the making of adjustments.

If adjustments are required they can be applied to either the input-output or heat rate curve. The effect of the adjustments on

the corresponding incremental rate curve will depend on the method of adjustment.

Adjustment of Input-Output Curves

Several methods are available for adjusting the input-output curve. If it is desired to have the incremental rate remain unaffected by the adjustment add to or subtract from the inputs a constant value. Mathematically this merely means that, if

$$y = f(x)$$

and

$$y' = f(x) + K$$

where K is a constant, then

$$\frac{dy'}{dx} = \frac{dy}{dx}$$

This is also demonstrated as follows:

Let O_1 and O_2 = successive values of output.
I_1 and I_2 = successive values of input corresponding to O_1 and O_2, respectively.
IR = the average incremental rate without adjustment of the input-output curve.
IR' = the average incremental rate for the adjusted input-output curve.
K = a constant.

Then the incremental output = $O_2 - O_1$, and the incremental input = $I_2 - I_1$, so that

$$IR = \frac{I_2 - I_1}{O_2 - O_1} = \frac{\Delta I}{\Delta O}$$

If the inputs are increased by a constant value K, then

$$IR' = \frac{(I_2 + K) - (I_1 + K)}{O_2 - O_1} = \frac{I_2 - I_1}{O_2 - O_1} = \frac{\Delta I}{\Delta O} = IR$$

If the inputs are multiplied or divided by a constant factor, the incremental rate values will have a corresponding change. Thus, if

$$y = f(x)$$

and

$$y' = K[f(x)]$$

then

$$\frac{dy'}{dx} = K\left(\frac{dy}{dx}\right)$$

Similarly

$$IR' = \frac{KI_2 - KI_1}{O_2 - O_1} = K\left[\frac{I_2 - I_1}{O_2 - O_1}\right] = K\left(\frac{\Delta I}{\Delta O}\right) = K(IR)$$

If the inputs are multiplied by a variable factor, the adjusted incremental rate curve will be a function of the slopes of the original input-output curve and the variable factor curve. Mathematically this means that, if

$$y = f_1(x)$$

$$K = f_2(x)$$

and

$$y' = K \cdot y$$

then

$$\frac{dy'}{dx} = K \cdot \frac{dy}{dx} + y \cdot \frac{dK}{dx}$$

The several ways in which a variable factor can be applied to the input-output curve are illustrated in Fig. 64. Curve 1 represents the com-

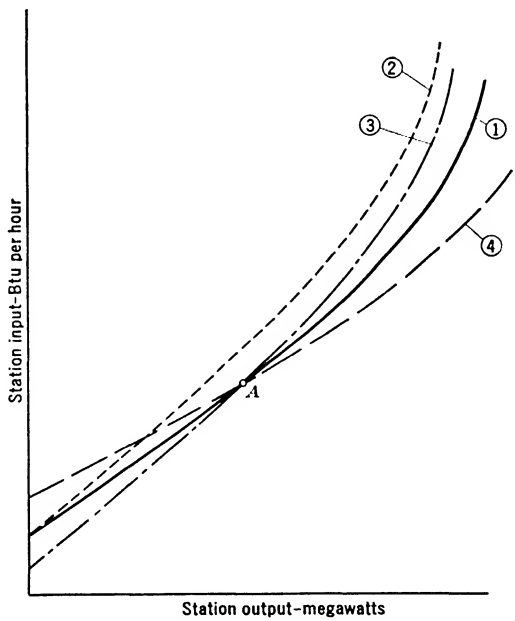


FIG. 64. Three methods of adjusting station input-output curves.

puted input-output curve; 2 is the adjusted input-output curve obtained by pivoting 1 about its minimum operating output; 3 is obtained by pivoting 1 about point A, which is usually the average load generated

during the operating period being checked, so that the inputs are reduced for loads less than the value at point A, and increased for greater loads; 4 shows the adjustment of the input-output curve by pivoting about point A in a direction opposite to that of curve 3 to give opposite effects.

For each of the methods discussed above, if the adjustments are properly made, the application of the adjusted inputs to the given load cycle should result in a computed overall station heat rate which is in close agreement with the actual heat rate. The corresponding adjusted incremental rate curves, however, will differ from each other.

Adjustment of Heat Rate Curves

The adjustments described above can also be applied to the computed heat rate curve. The addition or subtraction of a constant will change the incremental rate values by an amount equal to the constant. Thus, if

$$y = f(x)$$

represents the input-output curve, then

$$\frac{y}{x} = \frac{f(x)}{x}$$

represents the corresponding heat rate curve. The addition of a constant K to the original heat rate curve makes the adjusted heat rate curve

$$\frac{y'}{x} = \frac{y}{x} + K = \frac{f(x)}{x} + K$$

so that the adjusted input-output curve can be represented by

$$y' = y + K \cdot x$$

and

$$\frac{dy'}{dx} = \frac{dy}{dx} + K$$

where $\frac{dy'}{dx}$ = the adjusted incremental rate, and

$\frac{dy}{dx}$ = the original incremental rate.

Likewise, letting HR_1 and HR_2 = heat rates corresponding to the outputs O_1 and O_2, respectively,

$$I_1 = HR_1 \times O_1 \qquad \text{and} \qquad I_2 = HR_2 \times O_2$$

so that

$$IR = \frac{\Delta I}{\Delta O} = \frac{I_2 - I_1}{O_2 - O_1} = \frac{(HR_2 \times O_2) - (HR_1 \times O_1)}{O_2 - O_1}$$

and

$$IR' = \frac{(HR_2 \times O_2) - (HR_1 \times O_1) + K(O_2 - O_1)}{O_2 - O_1}$$

$$= \frac{I_2 - I_1}{O_2 - O_1} + K = \frac{\Delta I}{\Delta O} + K = IR + K$$

Multiplying or dividing the heat rate values by a constant or variable factor will change the incremental rates to the same extent as similar adjustments applied to the input values. Thus, for a constant factor, if

$$y = f(x)$$

then

$$HR = \frac{y}{x} = \frac{f(x)}{x}$$

and

$$HR' = \left(\frac{y}{x}\right) K = \frac{y'}{x}$$

so that

$$y' = K \cdot y \qquad \text{and} \qquad \frac{dy'}{dx} = K\left(\frac{dy}{dx}\right)$$

Similarly

$$IR' = \frac{(K \times HR_2 \times O_2) - (K \times HR_1 \times O_1)}{O_2 - O_1}$$

$$IR' = K\left(\frac{I_2 - I_1}{O_2 - O_1}\right) = K\left(\frac{\Delta I}{\Delta O}\right) = K(IR)$$

Incremental Production Costs

In developing the performance curves for a generating station, consideration was given only to its thermal performance, involving the station heat rates and incremental heat rates. For purposes of load division among stations of a system it is quite satisfactory to use the respective station incremental heat rates adjusted for differential fuel prices, since usually fuel represents the major item of production cost.

From a theoretical point of view the load division among stations should be based on their respective incremental production costs. Furthermore, this is the desired basis for billing energy interchanged between different companies. This requires that incremental cost curves be established for the production items other than fuel, namely, labor,

supplies, and maintenance. To be consistent, these should be established as functions of instantaneous demand or load. Unfortunately, there is no known method by which this can be done with a reasonable degree of accuracy. Hence it is common practice to resort to arbitrary methods of determining the incremental costs of labor, supplies, and maintenance.

Within certain limits, an increment of load can be generated without any change in the cost of labor, and in general the cost of labor will be influenced mostly by the amount of equipment in service. If it is necessary to operate additional boilers and turbine-generators to supply a given increment of load, obviously additional operating personnel will be required. The incremental labor costs will then be a function of the number of additional boilers and turbine-generators placed in service, and, within the limit of the additional capacity so provided, the incremental cost of labor will be fixed and independent of the incremental load supplied.

Agreements for the interchange of energy between systems frequently contain a provision for supplying firm or standby capacity which may involve incremental labor costs. It is preferable to incorporate the incremental labor costs with the charge for the standby capacity and to exclude them from the incremental energy charge.

Sometimes it becomes necessary to decide whether it would be more economical to supply an increment of load: (*a*) from a station already heavily loaded, or (*b*) from a station where additional equipment would have to be put in service. The decision should be made on the basis of the relative incremental production costs to supply the incremental load. In (*a*), probably no incremental labor costs would be involved; in (*b*), incremental labor costs should be included as a proper item of cost.

With respect to station supplies, analyses of the production costs for many stations indicate that the average unit cost of supplies per kilowatt-hour generated remains relatively constant. Hence no appreciable error is introduced by assuming the incremental cost of this item to be the same as the average unit cost. Further justification for this assumption is that this item is very small in comparison with other items of production cost.

Maintenance costs should include expenditures on boilers, turbine-generators, buildings, structures, and electrical, auxiliary, and miscellaneous equipment. It is generally recognized that boiler maintenance costs are influenced considerably by the steaming rates maintained. This is illustrated by the curve of Fig. 65, which shows boiler maintenance costs as a function of the coal burned for an installation consisting of stoker-fired boilers rated at 500 boiler horsepower each. The values

on both scales have been reduced to annual averages by dividing the annual costs and coal consumptions respectively by the annual boiler service hours.

For turbine-generators, the normal routine maintenance costs will generally depend on the service hours of the unit and not on the kilowatt-hour generation. Hence this item of maintenance is for all practical purposes not a function of the instantaneous generated load.

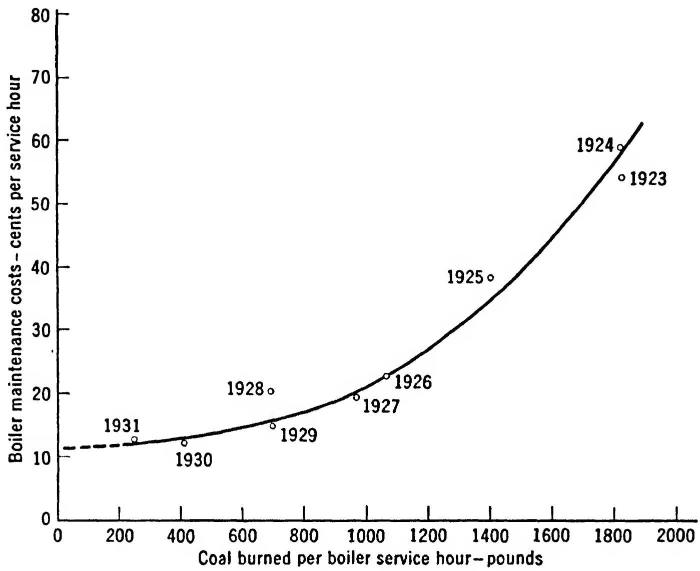


FIG. 65. Boiler maintenance cost shown as a function of the amount of coal burned.

Maintenance of the remaining power-plant equipment will usually have a fairly constant average unit cost per kilowatt-hour generated and it therefore may also be considered as the incremental cost.

In view of the fact that the determination of the incremental maintenance costs is at best but an "educated guess" there is hardly any justification for a breakdown of the costs among the classes of power-plant equipment mentioned above. Usually consideration need be given only to the maintenance costs for the station as a whole.

Among the arbitrary methods of determining the incremental maintenance costs, the most common one assumes that the incremental maintenance costs are some fixed percentage of the incremental fuel costs, which may be taken as the ratio of the annual maintenance costs to the annual fuel costs. As the annual fuel costs are easily determined from production cost statistics for the station, this method is a simple one to apply.

If station production cost statistics are available for a number of years during which the annual generation has a relatively large variation, the above method can be modified to permit the determination of the incremental maintenance costs in a more logical and consistent manner, as illustrated in Fig. 66.

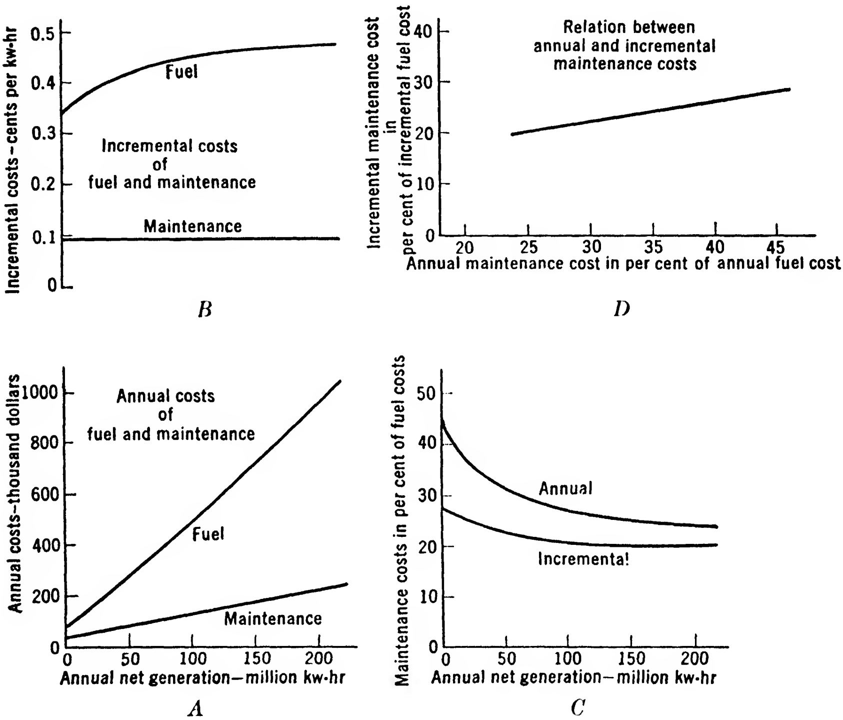


FIG. 66. Determination of incremental maintenance cost as a function of incremental fuel cost.

1. For the station in question, plot the annual fuel and maintenance costs as a function of the annual net generation. This is shown in Fig. 66*A* for a period covering seven years. The fuel costs should preferably be adjusted to some standard fuel price, and the maintenance costs should exclude any unusual item of repair and any item not a proper maintenance charge.

2. The slopes of the curves of Fig. 66*A* represent average incremental costs. These are plotted against annual generation as shown in Fig. 66*B*.

3. From the curves of Figs. 66*A* and 66*B* the ratio of the annual maintenance costs to the annual fuel costs and the ratio of the incremental maintenance costs to the incremental fuel costs, respectively, are de-

termined and plotted as shown in Fig. 66*C*. Referring to this figure it will be noted that the ratio of the annual costs is larger than the ratio of the incremental costs. The assumption that these are equal, which is the basis of the first method discussed above, is incorrect.

4. A cross plot from the curves of Fig. 66*C* permits the plotting of the ratio of the incremental costs against the ratio of the annual costs as shown in Fig. 66*D*. It is seen from this figure that, although the ratio of the annual costs varies from 24 to 45 per cent, the ratio of the incremental costs varies but from 20 to 28 per cent.

With a curve similar to that shown in Fig. 66*D* established it is a simple matter to derive the incremental production cost curve from the incremental heat rate curve. This is illustrated below for one value.

Incremental heat rate	12,000 Btu per kw-hr
Fuel price	18¢. per million Btu
Incremental fuel cost	0.216¢. per kw-hr
Ratio annual maintenance to annual fuel (from production cost statistics)	35 per cent
Ratio incremental maintenance to incremental fuel (from Fig. 66*D*)	24 per cent
Incremental fuel and maintenance costs (0.216 × 1.24)	0.268¢. per kw-hr
Incremental cost of supplies (from production cost statistics)	0.010¢. per kw-hr
Incremental production cost	0.278¢. per kw-hr

The most serious objection to the use of production cost statistics in the manner described above is that the maintenance costs for a given period are not necessarily the result of the generation during that period. For this reason a period covering less than one year should not be used. Averaging the costs over several years of operation is preferable, since any error resulting from deferred maintenance would thus be minimized. The fact remains that production cost statistics offer the only source of basic data for estimating incremental costs other than fuel, and hence must be used for whatever they are worth.

CHAPTER V

SYSTEM LOAD DIVISION

Integrated Input-Output Curves

Correct loading of generating stations has assumed increased importance as interconnections have become common, especially since the operation and control of boilers and turbine-generators now have reached a point where dependable station heat rate characteristics can be duplicated day after day. Application of the different methods of load division, however, is dependent essentially upon the nature of the data available. There has been a gradual transition, in varying degrees of accuracy, from the use of operating performance data on the basis of average monthly, weekly, or daily cost per kilowatt-hour to methods which permit of properly combining the economy characteristics of related equipment so that load can be divided on a minute-to-minute basis and full advantage can be taken of the immediate load conditions of the boiler room, turbine room, station, or system.

With the increased grouping of stations having quite different characteristics and the rapid introduction of new types of equipment, direct and accurate means of load division must be used to secure the best operating economy. In an interconnected group, individual stations do not necessarily follow the characteristic load curves of the system demand. Consequently there is a shifting of load factors and plant factors among the different stations so that the use of average input-output data has lost most of its value from the standpoint of correct allocation.

To illustrate, consider the curves of Fig. 67, which represent the characteristics of performance for two stations, taken from daily operating data. Each point represents the integrated 24-hour output for various station loads plotted against the total station heat input. Over a period of time a large number of values are obtained which give the average economy characteristic trend of the station. Straight lines are drawn through these points, and from this information a determination may be made as to how the stations should be loaded with respect to each other.

It is evident that, if a daily output of only 100,000 kw-hr is needed, station A could supply the load with less input than station B; if the output were 400,000 kw-hr, station B could supply it more economically

than station A. If both stations were required to be in operation, station A would be operated at minimum output until station B was fully loaded; and then station A would be loaded. This method of loading in no way takes advantage of the minute-to-minute economy relation between the stations, but merely utilizes the average integrated characteristics.

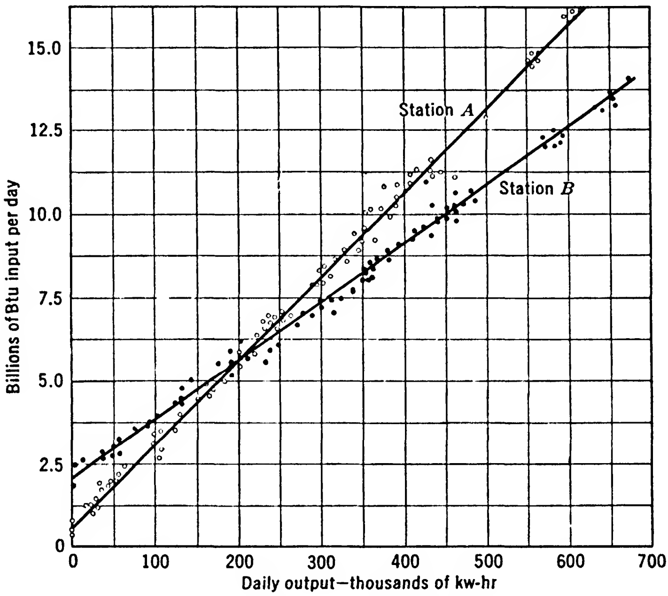


FIG. 67. Integrated input-output curves for two stations plotted from daily operating data.

The greatest objection to the use of this type of input-output curve is that the inputs are plotted as functions of the total output for some given period of time instead of as a function of the *instantaneous* load on the station. With a given combination of boilers and turbines in service, and for a given total output, the station input will depend upon the load cycle of the station. This is illustrated by the curves of Fig. 68. Given the station input-output curve of Fig. 68A, the average heat rate for a given period can vary between the values on the two curves of Fig. 68C as limits. For a given integrated output or plant factor, the average heat rate will then become a function of the load cycle. Thus, in Fig. 68B, six load cycles are shown, each of which will give the same integrated output corresponding to a plant factor of 50 per cent. The heat rates corresponding to each load cycle are shown by the numbered points of Fig. 68C.

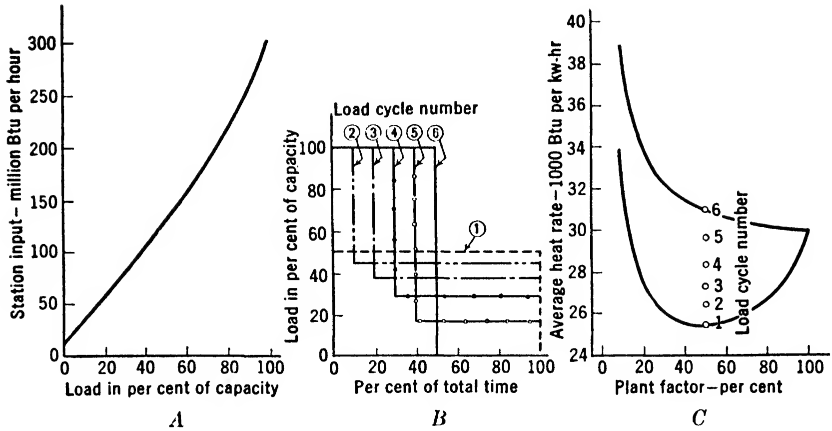


FIG. 68. Effect of load cycle on station heat rate at a constant plant factor.

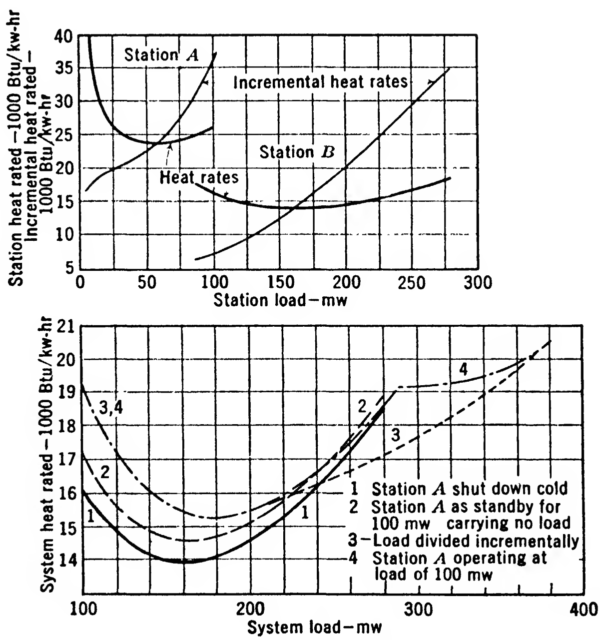


FIG. 69. Four methods of combining the operation of relatively low and high heat rate stations.

The significant point is that the slopes of the input-output curves of the form shown in Fig. 67 do not have any definite relationship to the slopes of the input-output curves plotted as functions of the instantaneous loads generated, and, as has previously been demonstrated, only instantaneous incremental values will define the proper division of load between two or more stations at any instant.

The effect of employing integrated input-output curves for load division purposes is illustrated by the curves of Fig. 69, which demonstrate four possible ways of operating two stations, one of which, A, has a high heat rate, and the other, B, has a low heat rate. If system conditions permit, the cold shutdown of the less efficient station A so that all the load may be supplied by B would give the best overall economy as shown by curve 1. Another possibility is that it may be necessary to operate station A with banked boilers to supply capacity for emergency use, but without being required to generate any load. The combined heat rate under these conditions is represented by curve 2. If in addition to providing boiler standby capacity it is necessary to provide spinning reserve at station A by operating one or more turbine-generators on the bus, then two alternatives are possible. One is to divide the load between the two stations incrementally giving overall performance as shown by curve 3; the other is to operate station A at minimum load until after station B is operating at capacity output (which would follow from the use of the integrated input-output curves), resulting in the combined heat rate curve 4. Curves 3 and 4 therefore indicate the relative economies from load division based on instantaneous and average integrated input-output curves, respectively.

Sequence of Adding Turbine-Generators

When a system contains several large generating stations with many turbine-generator units, the problem of selecting the correct combination of units for a given system load resolves itself into two parts:

1. Determination of the order in which units should be added to the line at each individual station, as the *station* load increases.

2. Determination of the order in which units should be put in service throughout the system, as the *system* load increases.

Since, at a given time, the same group of boilers will be used to supply steam to any combination of units in an individual station, the units should be added in the order of their relative efficiencies. Thus, as previously stated, to determine the proper combination at a given station load, it is only necessary to plot the turbine heat rate curves of successive combinations and select the one which gives the lowest *turbine* input. When several stations are operating in parallel, units put on the line at

one station must get their steam from the boilers of that station; hence the total *boiler* input of the system must be taken as the criterion for adding units and not the total turbine input. Calculation of the system input for several combinations will be required before the proper one can be ascertained for the given system load.

Computation of the system inputs for each of the numerous combinations, requiring a great deal of time and labor, can, however, be avoided by applying the following method:

1. For each individual station, compute the station input-output or heat rate curves corresponding to the proper sequence of adding units.

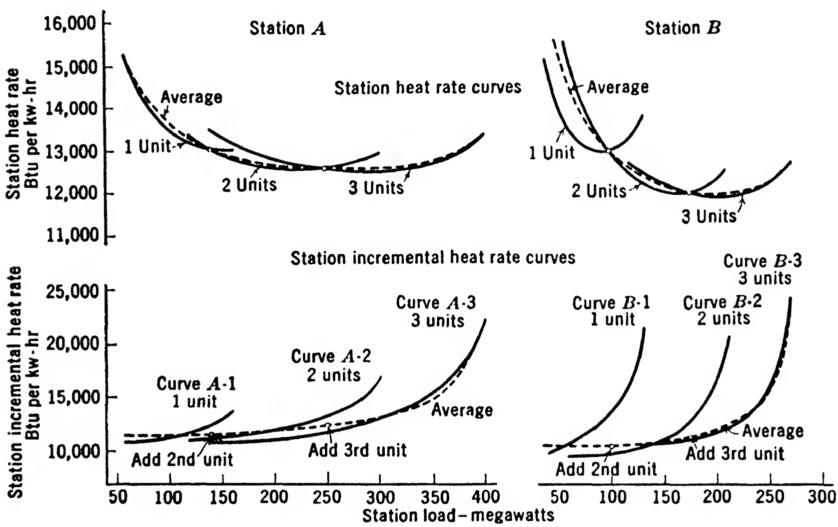


FIG. 70. Use of average heat rate curves to determine the sequence of adding turbine-generator units.

2. Plot the curves of step 1, and draw an average curve through them.

3. Compute and plot the average incremental rate curve corresponding to the average curve of step 2. On the incremental rate curve indicate the loads at which it is proper to add additional units.

4. Divide the system load on the basis of the average incremental rate curves of step 3. The station loads so determined will indicate the combination of units that should be operated at each station.

5. Redivide the system load using the *actual* incremental rate curves that correspond to the combinations determined under step 4.

For the purpose of illustration, curves covering the first three steps are shown in Fig. 70 for two stations with three turbine-generator units

in each. The determination of the proper combination of units and the corresponding load division between the stations are shown in Table XXXIV, following the procedure outlined as steps 4 and 5 above. Since

TABLE XXXIV

METHOD OF DETERMINING SEQUENCE FOR ADDING UNITS ON A SYSTEM

Load Division Using Average Incremental Heat Rate Curves					*Load Division Using Actual Incremental Heat Rate Curves*				
Load, Megawatts			Number of Units Indicated by Average Incremental Heat Rate Curves		Load, Megawatts			Curves Used for Load Division	
Total	Station *A*	Station *B*	Station *A*	Station *B*	Total	Station *A*	Station *B*	Station *A*	Station *B*
100	60	40	1	1	100	60	40	*A*-1	*B*-1
125	60	65	1	1	125	65	60	*A*-1	*B*-1
150	60	90	1	1	150	85	65	*A*-1	*B*-1
175	60	115	1	2	175	60	115	*A*-1	*B*-2
200	60	140	1	2	200	60	140	*A*-1	*B*-2
225	60	165	1	2	225	75	150	*A*-1	*B*-2
250	70	180	1	3	250	70	180	*A*-1	*B*-3
275	95	180	1	3	275	90	185	*A*-1	*B*-3
300	115	185	1	3	300	105	195	*A*-1	*B*-3
325	135	190	1	3	325	120	205	*A*-1	*B*-3
350	155	195	2	3	350	160	190	*A*-2	*B*-3
375	180	195	2	3	375	175	200	*A*-2	*B*-3
400	200	200	2	3	400	190	210	*A*-2	*B*-3
425	220	205	2	3	425	210	215	*A*-2	*B*-3
450	240	210	2	3	450	225	220	*A*-2	*B*-3
475	260	215	3	3	475	265	210	*A*-3	*B*-3
500	280	220	3	3	500	285	215	*A*-3	*B*-3
525	300	225	3	3	525	300	225	*A*-3	*B*-3
550	320	230	3	3	550	320	230	*A*-3	*B*-3
575	335	240	3	3	575	335	240	*A*-3	*B*-3
600	355	245	3	3	600	350	250	*A*-3	*B*-3
625	370	255	3	3	625	370	255	*A*-3	*B*-3
650	390	260	3	3	650	390	260	*A*-3	*B*-3
670	400	270	3	3	670	400	270	*A*-3	*B*-3

the method is an approximate one it is of interest to determine the magnitude of error introduced by its use, if any. This is illustrated by Table XXXV for a combined load of 300 megawatts. Using every possible combination of turbine-generator units, the load division was determined from the incremental heat rate curves of Fig. 70, corresponding to the respective unit combinations. It is to be noted that, for this load, the least overall input is obtained for the combination of units indicated by the use of the average incremental heat rate curves of the respective stations. Although this is not conclusive with respect to all values of sys-

tem load, experience with this method has shown that it will either indicate the exact combination of units that should be kept in service or else come so close thereto that the error involved will be a negligible factor.

TABLE XXXV

EFFECT OF TURBINE-GENERATOR COMBINATIONS ON OVERALL SYSTEM EFFICIENCY

Units in Service		Load, Megawatts			Heat Rate Btu per Kw-hr		Input, Million Btu per Hour		
Station *A*	Station *B*	Total	Station *A*	Station *B*	Station *A*	Station *B*	Station *A*	Station *B*	Total
1	2	300	130	170	13,140	12,000	1,708	2,040	3,748
1	3	300	105	195	13,450	11,950	1,412	2,330	3,742
2	1	300	215	85	12,580	13,000	2,705	1,105	3,810
2	2	300	145	155	13,020	12,020	1,888	1,863	3,751
2	3	300	120	180	13,240	12,000	1,589	2,160	3,749
3	1	300	230	70	12,680	13,250	2,916	928	3,844
3	2	300	155	145	13,300	12,100	2,062	1,755	3,817
3	3	300	140	160	13,500	12,150	1,890	1,944	3,834

Effect of Transmission-Line Losses

Economical division of load between two or more interconnected generating stations, when the tie-line losses are negligible, becomes merely a problem of operating the stations at loads which correspond to the same incremental rate value. This is the prevailing condition for metropolitan areas served by stations located at the load centers. For stations interconnected through long high-tension overhead lines, the losses are appreciable, and the load-division procedure should be modified to take them

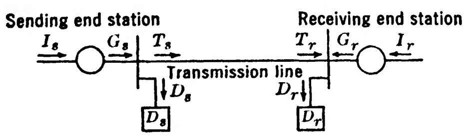


FIG. 71. Schematic diagram showing two generating stations connected by a transmission line.

into account. The theoretical principles involved can be understood more readily by considering the simplest example of two generating stations interconnected through a single transmission line.

A tie line may be treated as a machine in that the energy supplied to the line at the sending end may be considered as the input and the energy obtained at the receiving end, as the output. Hence, if the energy at the sending end is plotted as a function of the energy at the receiving end of the line, the relation between the input and output is established. The

slope of this input-output curve becomes the transmission-line incremental rate, and the reciprocal of the incremental rate is its incremental efficiency.

Referring to the schematic diagram of Fig. 71, let the subscripts s and r designate the sending and receiving ends of the transmission line, respectively. Then let

D_s and D_r = loads in the areas served by the respective generating stations (constants).
G_s and G_r = loads generated by the stations.
I_s and I_r = inputs to the generating stations corresponding respectively to G_s and G_r.
I_t = combined input to both generating stations.
T_s and T_r = loads on the transmission line at the sending and receiving ends, respectively.
R_s and R_r = station incremental rates corresponding respectively to the loads G_s and G_r.
E_{it} = incremental efficiency of the transmission line $= \dfrac{dT_r}{dT_s}$.

I_s, I_r, T_s, and T_r are assumed to be continuous functions with never-decreasing incremental rates. Then

$$I_t = I_s + I_r$$

$$G_s = D_s + T_s$$

$$G_r = D_r - T_r$$

Let G_s be the independent variable so that the conditions for minimum combined input to the two stations will be

$$\frac{dI_t}{dG_s} = 0 \qquad [68]$$

or

$$\frac{d(I_s + I_r)}{dG_s} = \frac{dI_s}{dG_s} + \frac{dI_r}{dG_s} = \frac{dI_s}{dG_s} + \left(\frac{dI_r}{dG_r} \times \frac{dG_r}{dG_s}\right) = 0 \qquad [69]$$

but

$$\frac{dI_s}{dG_s} = R_s; \quad \frac{dI_r}{dG_r} = R_r \qquad [70]$$

and

$$\frac{dG_r}{dG_s} = \frac{d(D_r - T_r)}{d(D_s + T_s)} = -\frac{dT_r}{dT_s} = -E_{it} \qquad [71]$$

since D_s and D_r are constants.

Substituting equations 70 and 71 in equation 69,

$$R_s - (R_r \times E_{it}) = 0 \quad [72]$$

or

$$R_s = R_r \times E_{it} \quad [73]$$

and

$$R_r = R_s \times \frac{1}{E_{it}} \quad [74]$$

Equations 73 and 74 modify the general rule that stations should be operated at outputs corresponding to the same incremental rate value, in that a correction factor is included to compensate for the transmission-line losses. The correction factor is the incremental efficiency of the tie line corresponding to the load being transmitted and may be applied to the incremental rate of the station at either the receiving or the sending end of the line.

Load Division between Two Interconnected Stations

To illustrate the application of equations 73 and 74, recourse will be had to the simple example of two interconnected generating stations

TABLE XXXVI

SAMPLE CALCULATION SHOWING METHOD OF ADJUSTING INCREMENTAL HEAT RATES OF STATION A FOR TIE-LINE LOADS

Tie Line Load T_A	Demand in Area Served by Station D_A	Net Load Generated G_A	Actual Station Incremental Rate R_A Btu/Hr/Kw	Tie-Line Incremental Efficiency E_{it}	Adjusted Station Incremental Rate R'_A Btu/Hr/Kw
+100	400	500	16,000	0.800	20,000
+ 80	400	480	15,230	0.840	18,130
+ 60	400	460	14,710	0.880	16,720
+ 40	400	440	14,280	0.920	15,520
+ 20	400	420	13,880	0.960	14,460
0	400	400	13,500	1.000	13,500
− 20	400	380	13,150	0.958	12,600
− 40	400	360	12,830	0.916	11,750
− 60	400	340	12,530	0.871	10,910
− 80	400	320	12,270	0.825	10,120
− 90	400	310	12,160	0.800	9,730

When T_A is +, tie line feeds out. $R'_A = R_A \times \frac{1}{E_{it}}$

When T_A is −, tie line feeds in. $R'_A = R_A \times E_{it}$

All loads in megawatts.

whose incremental rate curves are shown in Fig. 72 as those for station A and station B. The losses and incremental efficiencies of the tie line are shown respectively in Figs. 73 and 74. The incremental rate curves de-

termining the proper load division between the two stations are shown in Fig. 75.

Since two stations are involved, adjustments for the tie-line losses may be made to the incremental rate curve of either station. Referring

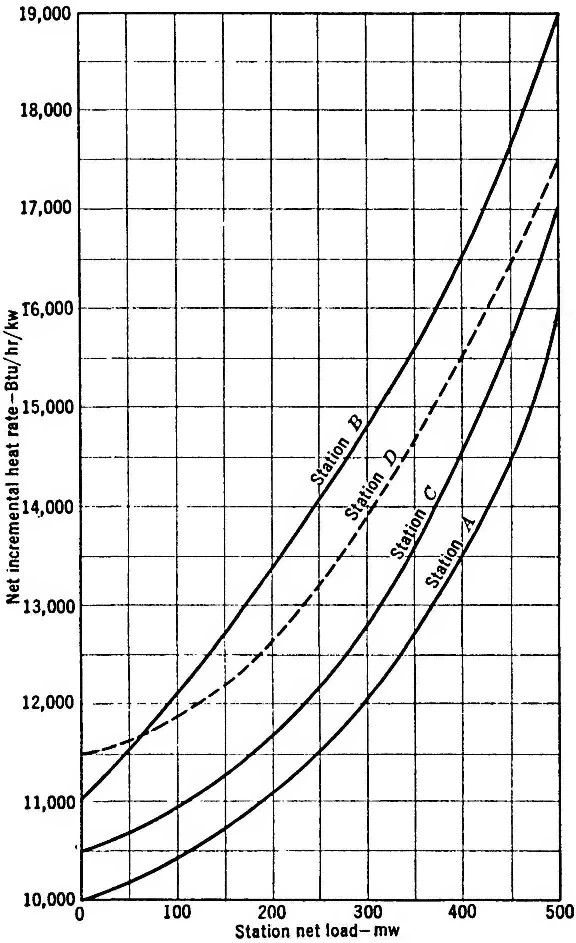


FIG. 72. Incremental heat rate curves for interconnected stations.

to the left-hand section of Fig. 75, the curve net represents the incremental performance of station A adjusted for the line losses. It is to be noted that these are plotted with the local demand D_A supplied by the station as the parameter. A sample calculation of one curve for the parameter $D_A = 400$ is shown in Table XXXVI. Referring to this table, since adjustments are made to the incremental rate curve for station A, the

numerical sign of the tie-line load indicates whether the station is receiving energy from the line or supplying energy to it. This distinction is important with reference to the adjustment of the incremental rates of

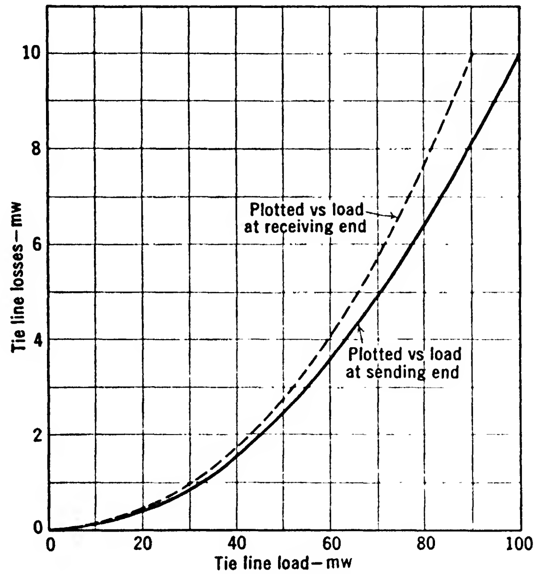


FIG. 73. Transmission line loss curve.

the station. A plus sign designates that the station is supplying energy to the line, and the adjustment is made by *dividing* the station incremental rate by the incremental efficiency of the tie line corresponding to the load supplied to it. A minus sign designates that the station is receiving energy from the line, and the adjustment is made by *multiplying* the station incremental rate by the incremental efficiency of the tie line.

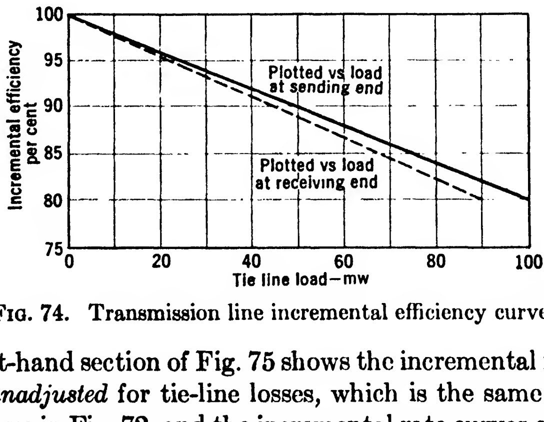


FIG. 74. Transmission line incremental efficiency curves.

The right-hand section of Fig. 75 shows the incremental rate curve for station *B*, *unadjusted* for tie-line losses, which is the same as the corresponding curve in Fig. 72, and the incremental rate curves combining the

curve for station B with each of the adjusted curves for station A. The combined incremental rate curves are plotted against the total demands

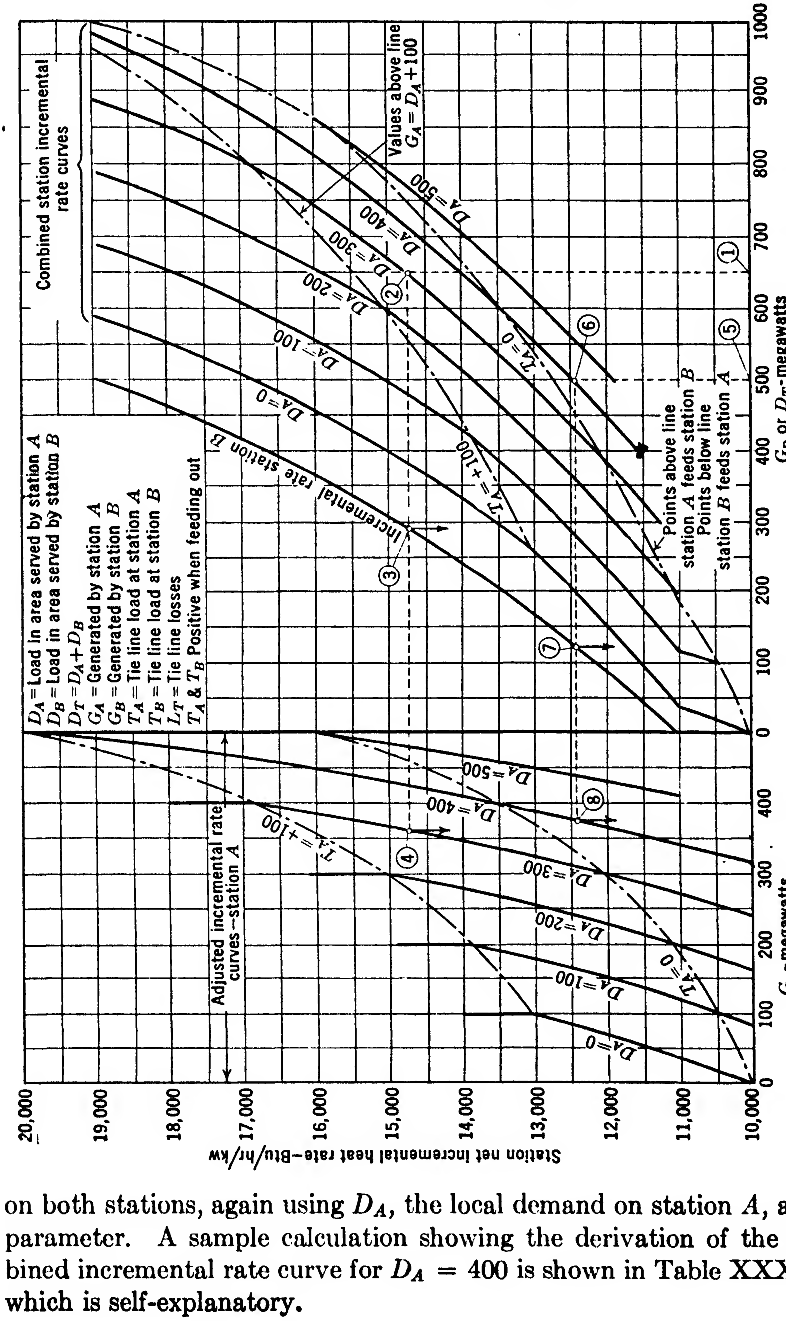


FIG. 75. Incremental heat rate curves used to determine the loading of two interconnected generating stations.

on both stations, again using D_A, the local demand on station A, as the parameter. A sample calculation showing the derivation of the combined incremental rate curve for $D_A = 400$ is shown in Table XXXVII, which is self-explanatory.

TABLE XXXVII

SAMPLE CALCULATION SHOWING DERIVATION OF COMBINED INCREMENTAL HEAT RATES FOR STATIONS A AND B

Incremental Heat Rate Btu/Hr/Kw R'_A or R_B	Net Loads Generated		Demand in Area Served by Station A D_A	Tie-Line Load T_A	Tie-Line Losses L_T	Total Load Generated $G_A + G_B$	Total Demand $D_A + D_B$
	Station A G_A	Station B G_B					
20,000	500	500	400	+100	10.0	1,000	990.0
19,500	496	500	400	+ 96	9.2	996	986.8
19,000	490	500	400	+ 90	8.1	990	981.9
18,500	484	482	400	+ 84	7.1	966	958.9
18,000	478	463	400	+ 78	6.1	941	934.9
17,500	472	442	400	+ 72	5.2	914	908.8
17,000	464	419	400	+ 64	4.1	883	878.9
16,500	457	395	400	+ 57	3.3	852	848.7
16,000	448	368	400	+ 48	2.3	816	813.7
15,500	440	339	400	+ 40	1.6	779	777.4
15,000	430	309	400	+ 30	0.9	739	738.2
14,500	420	277	400	+ 20	0.4	697	696.6
14,000	410	242	400	+ 10	0.1	652	651.9
13,500	400	207	400	0	0	607	607.0
13,000	389	169	400	− 11	0.1	558	557.9
12,500	378	128	400	− 22	0.5	506	505.5
12,000	366	87	400	− 34	1.2	453	451.8
11,500	354	44	400	− 46	2.3	398	395.7

$D_A + D_B = G_A + G_B - L_T.$

Tie line feeding out, T_A is +.
Tie line feeding in, T_A is −.
All loads in megawatts.

TABLE XXXVIII

DATA FOR EXAMPLES SHOWN IN FIG. 75

	Megawatts			
	Example 1		Example 2	
D_T		650		500
D_A		300		400
D_B		350		100
G_B		292		124
G_A		362		377
T_A	(362 − 300)	+62	(400 − 377)	−23
T_B	(350 − 292)	−58	(124 − 100)	+24
L_T	(62 − 58)	4	(24 − 23)	1

Two examples illustrating the use of the curves of Fig. 75 are indicated in this figure; the conditions upon which these examples are based are given in Table XXXVIII. In example 1, the diagram is entered at point 1. At point 2 the incremental rate is indicated, the value of which corresponds both to the adjusted value for station *A* and the actual value for station *B*. The intersections of the curves of stations *B* and *A* at points 3 and 4, respectively, with the horizontal line at this incremental rate value, determine the loads that should be generated by the respective stations.

Similarly, for example 2, the diagram of Fig. 75 is entered at point 5 and the respective station loads determined at points 7 and 8.

Load Division between More than Two Interconnected Stations

When the interconnection consists of more than two stations, an exact solution requires the computation of a very large number of curves involving a prohibitive amount of labor. An approximate method, however, gives results in very close agreement with those obtained by an exact solution. Briefly, the method involves three steps:

1. Determination of the load division neglecting line losses.
2. Approximation of the line losses.
3. Redivision of the load increased by the approximate line losses.

TABLE XXXIX

EXAMPLE ILLUSTRATING THE USE OF FIG. 76

Station	Demand in Area Served by Station	Step 1			Step 2		
		Net Load Generated	Tie-Line Load	Tie-Line Losses	Net Load Generated	Tie-Line Load	Tie-Line Losses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	300	355	+55	3.1	357	+57	3.3
B	200	222	+22	0.5	224	+24	0.6
C	400	395	− 5	0	396	− 4	0
D	400	328	−72	6.0	332	−68	5.4
Totals	1,300	1,300		9.6	1,309		9.3

Column 1. The demands in the areas served are known.

2. The total demand is divided neglecting line losses. The station loads are obtained from points 1 to 4, inclusive, on line *A–A*. The position of line *A–A* is determined by cut-and-try method so that the sum of the station loads equals 1,300.
3. The tie-line loads are approximate. Positive flow is toward the common point *N*.
4. The line losses are obtained from Fig. 73.
5. The station loads are determined from points 5 to 8, inclusive, on line *B–B*, the position of which is determined by cut-and-try method until the sum of the station loads equals 1,309.
6. Revised tie-line loads.
7. Tie-line losses corresponding to revised tie-line loads.

All loads in megawatts.

Application of this method to a hypothetical interconnection consisting of four stations is illustrated by Fig. 76 and Tables XXXIX and XL.

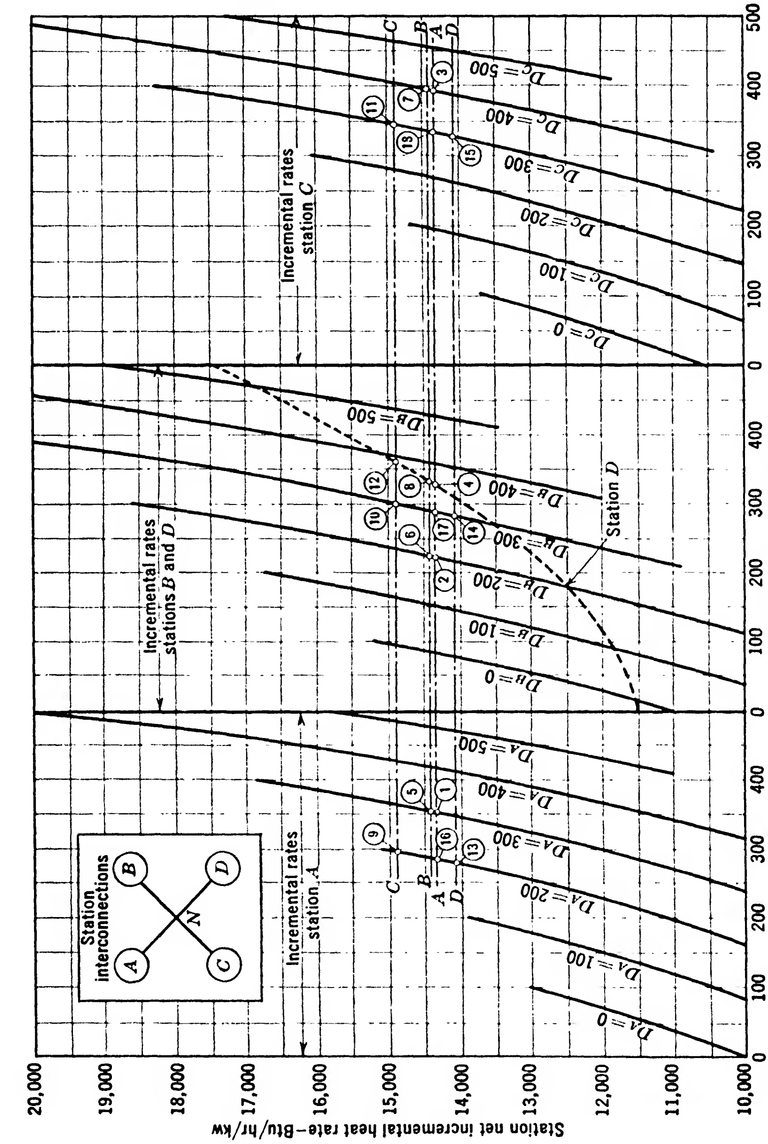


FIG. 76. Incremental heat rate curves used to determine the loading of four interconnected generating stations.

For the sake of simplicity, the stations were assumed to be of equal capacity, and interconnected as shown in Fig. 76. The incremental rate curves for the respective stations are shown in Fig. 72. The tie-line losses and incremental efficiencies, shown respectively in Figs. 74 and 75, were

TABLE XL

EXAMPLE ILLUSTRATING THE USE OF FIG. 76

Station	Demand in Area Served by Station	Step 1			Step 2			Step 3		
		Net Load Generated	Tie-Line Load	Tie-Line Losses	Net Load Generated	Tie-Line Load	Tie-Line Losses	Net Load Generated	Tie-Line Load	Tie-Line Losses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	200	296	+ 96	....	280	+ 80	6.4	286	+ 86	7.4
B	300	300	0	....	282	− 18	0.3	288	− 12	0.2
C	300	344	+ 44	....	328	+ 28	0.8	334	+ 34	1.1
D	500	360	−140	....	410	− 90	10.0	410	− 90	10.0
Totals	1,300				1,300		17.5	1,318		18.7

Step 1. Procedure same as outlined in Table XXXIX. Station loads determined from points 9 to 12, inclusive, on line *C–C*. Line losses not determined since overload is indicated on line from station *D*.

Step 2. The minimum generation of station *D* to eliminate overloading the tie line is 500 − 90 = 410 megawatts, since 90 megawatts is the maximum load that can be supplied to the station. Neglecting line losses, stations *A*, *B*, and *C* must generate 1,300 − 410 = 890 megawatts. Points 13, 14, and 15 on line *D–D* indicate the three station loads totaling to 890 megawatts.

Step 3. Procedure is the same as in step 2 of Table XXXIX except that the load on station *D* is kept at 410 megawatts. Total generation by stations *A*, *B*, and *C* is 1,318 − 410 = 908 megawatts, and respective station loads are indicated by points 16, 17, and 18 on curve *A–A*.

All loads in megawatts.

assumed to apply for each section of the interconnection between each station and the common point at N of Fig. 76.

Since four stations are involved, the incremental rates of three stations, A, B, and C, must be adjusted, and these are shown in Fig. 76 plotted with the respective local demands as parameters. The incremental rate curve of station D is shown as unadjusted for tie-line losses.

In Table XXXIX the final load division corresponds to a total demand of 1,299.7 megawatts compared with the actual demand of 1,300 megawatts; in Table XL the demand corresponding to the load division is 1,299.3 megawatts compared with 1,300 megawatts. In both, the differences are negligible for all practical purposes and well within the accuracy of the metering facilities and the accuracy with which the performance curves of the generating stations can be established.

Incremental Tie-Line Losses

In considering the transmission-line losses, it was assumed that these could be presented graphically as a function of the load at either the sending or receiving end of the line. Actually, for a given load at either end of the line, the losses will be a function of the power factor. In addition to the line losses, consideration must be given to the losses of transformers forming part of the transmission circuit, and to the losses of synchronous condensers when in operation. These may be grouped with the line losses to constitute the total losses.

Only the I^2R losses need be considered. The resistance R is fixed by design so that the losses are a function of the square of the current, I^2. For short transmission lines, say less than 40 miles, the charging current of the line may be neglected, so that the current at the receiving and at the sending end may be considered to be the same in value. By reason of this assumption, a simple relationship can be established between the line losses, incremental line losses, and incremental rate or incremental efficiency of a short transmission line, the derivation of which follows:

Let R = line resistance per wire, in ohms.
I = line current, in amperes.
kva = total three-phase transmitted kilovolt-amperes.
kw = total three-phase transmitted energy (T_r).
$kvar$ = total three-phase transmitted reactive kilovolt-amperes.
kv = line voltage, in kilovolts.
P.F. = power factor of the transmitted energy.
T_L = total three-phase line losses, in kilowatts.
T_{La} = three-phase line losses due to transmitted energy (T_r), in kilowatts.

T_{Lr} = three-phase line losses due to transmitted reactive kilovolt-amperes, in kilowatts.

Then

$$T_L = \frac{3I^2R}{1{,}000} \qquad [75]$$

but

$$I = \frac{kva}{\sqrt{3}kv} \qquad [76]$$

and

$$I^2 = \frac{(kva)^2}{3(kv)^2} \qquad [77]$$

Therefore

$$T_L = \frac{3R}{1{,}000} \times \frac{(kva)^2}{3(kv)^2} = \frac{R(kva)^2}{1{,}000(kv)^2} \qquad [78]$$

Since

$$(kva)^2 = (kw)^2 + (kvar)^2$$

$$T_L = \frac{R}{1{,}000(kv)^2}(kw)^2 + \frac{R}{1{,}000(kv)^2}(kvar)^2 \qquad [79]$$

Let

$$K = \frac{R}{1{,}000(kv)^2}$$

then

$$T_L = K(kw)^2 + K(kvar)^2 \qquad [80]$$

where

$$T_{La} = K(kw)^2 \qquad [81]$$

and

$$T_{Lr} = K(kvar)^2 \qquad [82]$$

so that

$$T_L = T_{La} + T_{Lr} \qquad [83]$$

From equation 78 the line losses can be plotted as a function of the total kilovolt-amperes transmitted. The component of line losses due to the energy transmitted can be plotted as a function of the kilowatts from equation 81, and similarly the component of line losses due to the reactive kilovolt-amperes transmitted can be plotted as a function of the reactive kilovolt-amperes from equation 82. The total losses T_L and each of the component losses T_{La} and T_{Lr} can be represented by a single curve by plotting the losses against a common abscissa scale. This is illustrated in Fig. 77A for a 66-kv line having a resistance of 5.70 ohms per phase. For a transmitted load of 16,000 kw at 0.8 P.F., the calculated losses from equations 78, 81, and 82 would be as follows:

$$kw = 16{,}000$$

$$\text{P.F.} = 0.8$$

$$kva = \frac{16{,}000}{0.8} = 20{,}000$$

$$kvar = 20{,}000 \times 0.6 = 12{,}000$$

$$K = \frac{5.70}{1{,}000(66)^2} = 131 \times 10^{-8}$$

$$T_L = 131 \times 10^{-8}(20{,}000)^2 = 524 \text{ kw}$$

$$T_{La} = 131 \times 10^{-8}(16{,}000)^2 = 335 \text{ kw}$$

$$T_{Lr} = 131 \times 10^{-8}(12{,}000)^2 = 189 \text{ kw}$$

or

$$T_L = T_{La} + T_{Lr} = 335 + 189 = 524 \text{ kw}$$

The above losses can be obtained directly from the curve of Fig. 77*A* by the use of the proper abscissa values.

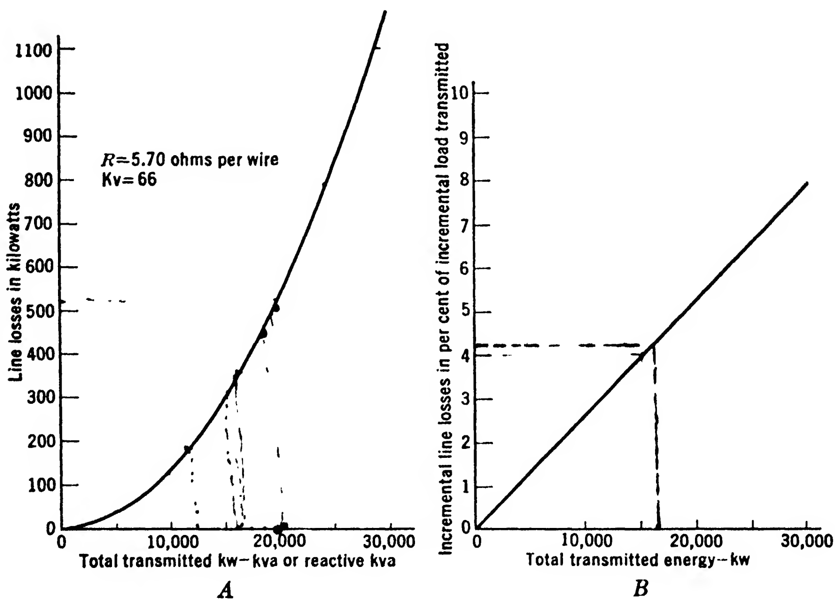


FIG. 77. *A*. Transmission line losses. *B*. Transmission line incremental losses.

For long transmission lines, say more than 40 miles, the above procedure should not be applied, for, by neglecting the effect of the charging current, an appreciable error may be introduced.

With the line losses established, the relationship between the loads at the sending and receiving ends of the line becomes

$$T_s = T_r + T_L \tag{84}$$

But

$$R_{it} = \frac{dT_s}{dT_r} = 1 + \frac{dT_L}{dT_r} \tag{85}$$

also

$$\frac{dT_L}{dT_r} = 2K(kva)\frac{d(kva)}{dT_r} \tag{86}$$

and

$$kva = \frac{T_r}{\text{P.F.}}$$

so that

$$\frac{d(kva)}{dT_r} = \frac{\text{P.F.} - T_r\left[\dfrac{d(\text{P.F.})}{dT_r}\right]}{(\text{P.F.})^2}$$

Therefore

$$R_{it} = 1 + 2K\frac{T_r}{\text{P.F.}}\left[\frac{\text{P.F.} - T_r\dfrac{d(\text{P.F.})}{dT_r}}{(\text{P.F.})^2}\right] \tag{87}$$

Equation 87 defines the incremental rate of the transmission line, R_{it}, as a function of the transmitted load T_r, the power factor P.F., and the rate of change of the power factor $d(\text{P.F.})/dT_r$ with respect to the transmitted load. This equation applies to any transmission line, and for its application it is necessary to establish the power factor and rate of change of the power factor as a function of the transmitted load T_r. If the power factor is plotted as a function of the transmitted load, then $d(\text{P.F.})/dT_r$ is the slope of this curve.

If energy is transmitted at constant power factor, then

$$\frac{d(kva)}{dT_r} = \frac{1}{\text{P.F.}} \tag{88}$$

and

$$R_{it} = 1 + \left[2K \times \frac{T_r}{(\text{P.F.})^2}\right] \tag{89}$$

Equation 89 can readily be applied to establish the transmission-line incremental rate curve for any given power factor. To facilitate the calculations, the incremental line losses should be established as a function of the transmitted load at unity power factor. This is illustrated in Fig. 77*B* for the same line represented by the curve of Fig. 77*A*. It is to be

noted that the incremental line losses will plot as a straight line passing through the origin. Hence it is necessary to compute only one point. To illustrate the use of Fig. 77, consider the following example:

Transmitted load (T_r)	16,000 kw
Power factor of transmitted load	0.80
Total kva transmitted (16,000 ÷ 0.8)	20,000 kva
Total line losses from Fig. 77*A* @ 20,000 kva	524 kw
Load at sending end of the line (16,000 + 524)	16,524 kw
Incremental line losses at unity power factor from Fig. 77*B* for T_r = 16,000 kw	0.0419
Incremental line losses at 0.80 power factor [0.0419 ÷ $(0.80)^2$] =	0.0655
Incremental rate of transmission line (1 + 0.0655)	1.0655
Incremental efficiency of transmission line (1 ÷ 1.0655)	0.939

Load Division between Hydroelectric Units

The fundamentals of economy loading apply to hydro as well as to steam plants. Hydro plants require no steam-generating equipment or

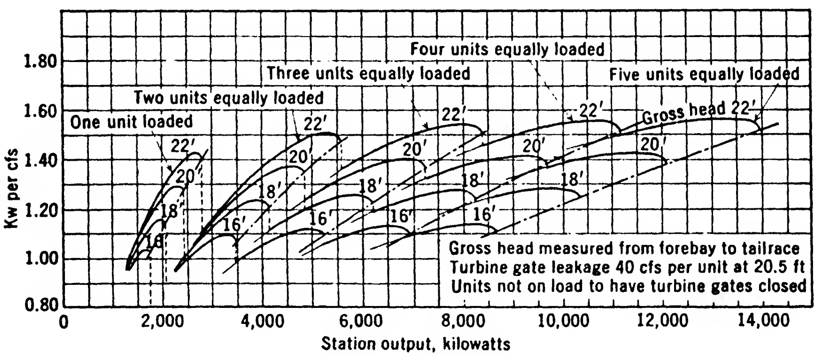


FIG. 78. Performance curves for a low-head hydroelectric power plant.

steam condensers. Production costs are lower since fuel costs are eliminated and the requirements for operating labor and maintenance are considerably reduced.

According to E. B. Strowger,[1] hydro plants may be classified with respect to the head at which the plants operate. Figure 78 illustrates the performance of a plant operating at a head which varies from 16 to 22 feet. The units in a station operating at such low heads are generally provided with individual penstocks and the station is built integral with

[1] "How We Raise Hydro Efficiency" by E. B. Strowger, *Electrical World*, April 14, 1934.

the dam. It is to be noted that, for this type of plant, improvement in efficiency is obtained as units are added to those already in operation for the reason that the leakage loss in the standby units is either reduced or eliminated.

The curves of Fig. 79 are typical for a plant operating at relatively high heads. The plant is supplied with water through a long pipe line introducing a conduit loss or loss of head available to the turbines which varies as the square of the velocity. As additional units are added, the

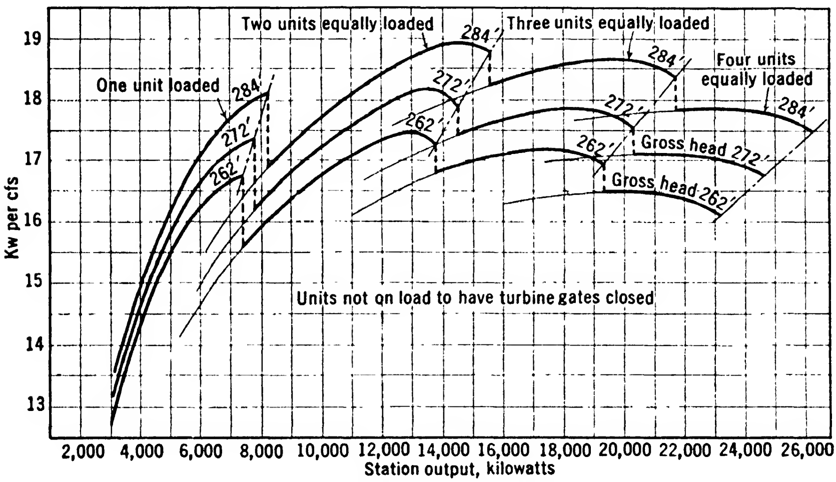


FIG. 79. Performance curves for a high-head hydroelectric power plant.

leakage loss is reduced but the conduit loss is increased, so that a point is reached where the increased conduit loss is more than enough to offset the gain due to the decrease in leakage loss. Hence, as shown by the curves of Fig. 79, utilization of the water at the highest efficiency is obtained at this plant when only two units are in operation.

If the performance characteristics of the several units in a station differ, the proper load division should be incrementally determined. Figure 80 illustrates typical performance curves for a hydro unit. These are plotted as functions of the generator output, and they include:

1. The efficiency curve which indicates the kilowatt load developed per cubic foot per second flow.

2. The input-output curve which indicates the total rate of flow in cubic feet per second.

3. The incremental rate curve which indicates the change in the rate of flow required per kilowatt change in load.

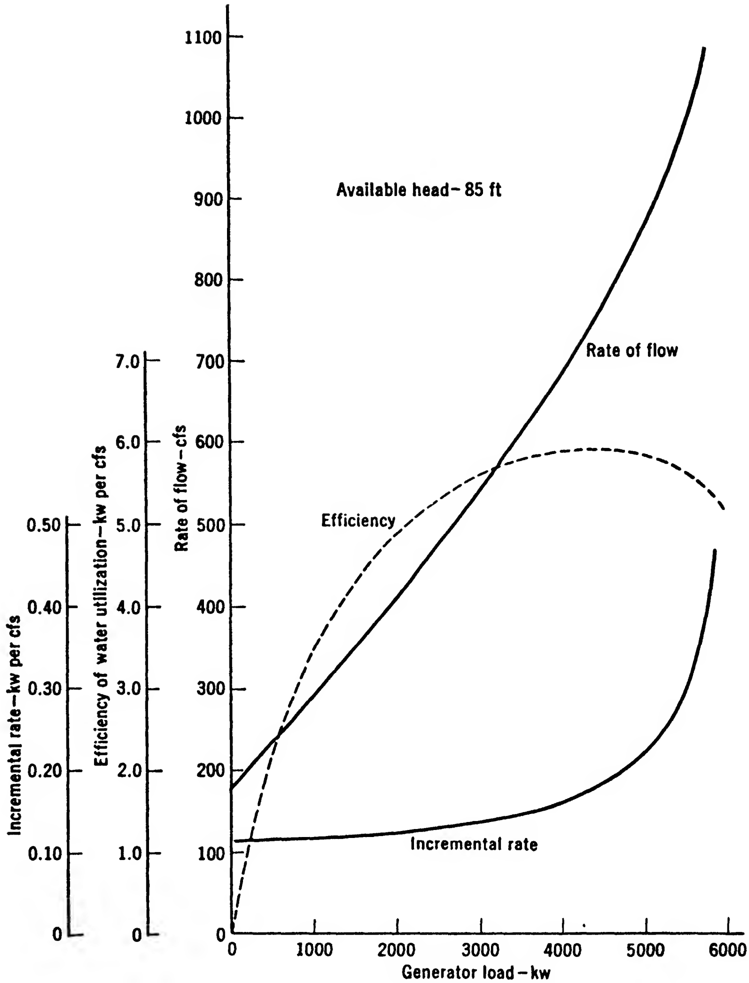


FIG. 80. Performance curves for a hydroelectric turbine-generator.

Load Division between Steam and Hydro Plants

When a system consists entirely of hydro plants, the efficient utilization of the water is not relatively important, unless the supply is limited with respect to requirements or there is an unlimited market for the energy which can be produced. In a combination of steam and hydro plants, the hydro plants should be operated to obtain the best utilization of the available water. This does not necessarily imply the most efficient operation of the hydro equipment, but such use as will result in the least overall production costs, taking into account the effect of hydro genera-

tion on the production costs of the steam plant. In other words, hydro generation should not be favored at the expense of steam generation.

Each system will undoubtedly have problems which are peculiar to that system. The reader is referred to an article by Estrada and Finlaw [1] which describes how efficient utilization of hydro capacity is obtained for the particular conditions prevailing in the Philadelphia Electric Company system.

Since there are no fuel costs in the production of hydroelectric energy, incremental energy can be supplied by a hydro plant at practically no incremental cost. It follows, therefore, that, with respect to load allocation between steam and hydro plants, the generation at the steam plants should be displaced by available generation at the hydro plants, so that the maximum decrement production costs will obtain at the steam plants. The extent to which this can be accomplished depends on the nature of the daily load curve for the system, the type of hydro plants, and the available water. Hydro plants may be operated to supply the system base load, peak load, or both. In a run-of-river plant, the flow of the river must be utilized as it comes, so that this plant would normally supply the system base load. Plants with storage or pondage facilities may be operated to supply either base or peak load, depending on how the flow may best be utilized.

To illustrate how the system load should be allocated properly between steam and hydro plants, a hypothetical system will be assumed consisting of three plants of which one is steam and two are hydro. The steam plant consists of two units with a capacity of 25 megawatts each; a run-of-river hydro plant consists of three units rated at 20 megawatts each; and a reservoir plant of two units rated at 15 megawatts each. The daily load cycle of the system is represented by the curve of Fig. 81*A*. The run-of-river plant can develop 40 megawatts of load continuously from the available stream flow. The reservoir plant can develop 47 megawatt-hours per day from the available storage.

The block of system load which each plant should supply is indicated by the respective cross-hatched areas under the load curve of Fig. 81*A*. These were obtained by using the energy-load curve of Fig. 81*B*, derived from the load curve of Fig. 81*A*, which indicates the instantaneous demand as a function of the time of day. Hence the total area under the daily load curve represents the total energy corresponding to the instantaneous demands during the 24-hour period. The energy-load curve shows the relation between a given demand and the energy corresponding thereto. Thus point 1 on the energy-load curve indicates an energy value

[1] "Allocation of Loads to Conowingo Hydro Station" by Herbert Estrada and John Finlaw, *Electrical World*, May 12, 1934.

which corresponds to the area of block A under the daily load curve; point 2 corresponds to the combined areas of blocks A and B; and point 3 corresponds to the entire area under the daily load curve.

Since the energy available from the reservoir plant is limited, it can best be utilized to carry the system peak load. The energy requirement of the system is 1,480 megawatt-hours, of which 47 megawatt-hours is to

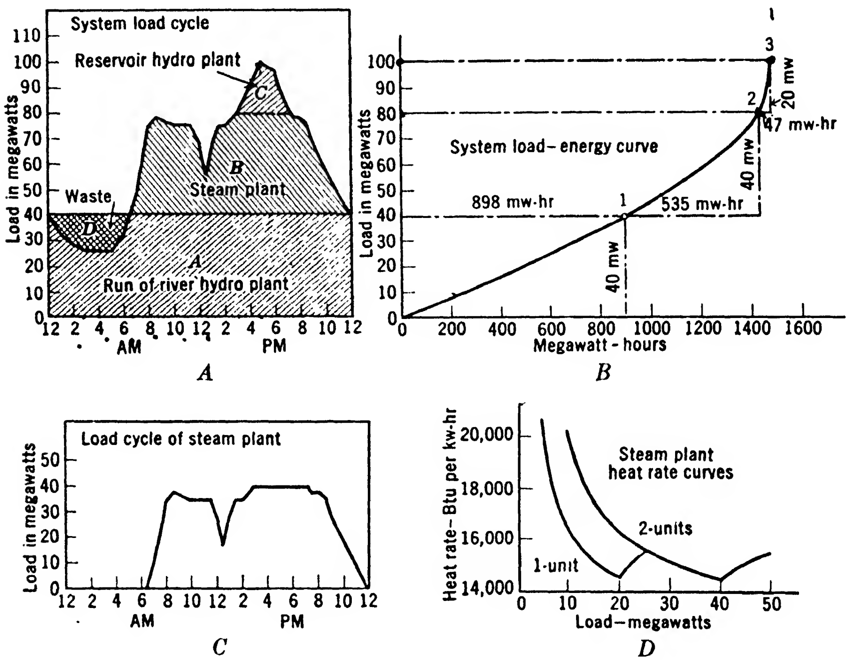


FIG. 81. Load distribution among one steam and two hydroelectric power plants.

be supplied by the reservoir plant. The remaining two plants must therefore supply 1,480 − 47 or 1,433 megawatt-hours. This value corresponds to point 2 on the energy-load curve, which locates the base of block C under the daily load curve at a value corresponding to a demand of 80 megawatts, so that the reservoir plant is operated to supply all loads in excess of a system load of 80 megawatts.

The run-of-river plant, not being provided with storage facilities, must utilize the stream flow as it comes; otherwise the energy available therefrom will be wasted. Hence this plant should provide the system base load of 40 megawatts which is represented by point 1 on the energy-load curve and fixes the boundaries of block A under the daily load curve. It is to be noted, however, that the system demand is less than that available from the stream flow from midnight to approximately 6:30 A.M., so

that the energy corresponding to block D would be wasted unless an interconnection were available for transmission to some other system.

The steam plant would then supply the remainder of the system load represented by the area of block B, this being equivalent to the difference between the values at points 1 and 2 on the energy-load curve. The load curve of the steam plant is shown in Fig. 81C. It is seen that this plant, with two units in service, would operate at loads ranging from 35 to 40 megawatts, which are at or near the most efficient point of its corresponding heat rate curve shown in Fig. 81D.

The statistics relating to the operation of the three plants are shown in Table XLI.

TABLE XLI

SUMMARY OF OPERATING DATA FOR SYSTEM SHOWN IN FIG. 81

	Steam Plant	Hydro Plants		Total System
		Run of River	Reservoir	
Generation, mw-hr	535	898	47	1,480
Maximum demand, mw	40	40	20	100
Total service hours	17.6	24	4.3	24
Load factor, per cent	76.0	93.6	54.6	61.7
Limits of system load supplied, mw	40–80	0–40	80–100	

CHAPTER VI

MISCELLANEOUS APPLICATIONS

Industrial Turbine-Generators

Industrial turbine-generators embrace a variety of types. Under a broad classification they may be designated as condensing or non-condensing, the latter being referred to as back-pressure turbines. If steam is extracted at one or more points the turbine is classed as an extraction or bleeder-type turbine. Extraction turbines may be further classified as automatic or non-automatic. In the automatic ones the extracted steam is maintained at constant pressure; in the non-automatic, the extraction pressure (above exhaust) varies in approximately direct proportion to the quantity of steam flowing past the point of extraction. In addition to the above, there is the mixed-pressure turbine which is designed for the admission of steam at reduced pressures to one or more lower stages.

Regardless of the type of turbines involved, the principles of load division to be applied are the same. Essentially, the problem involves the division of the electric load and quantity of steam extracted from the respective turbines in such a manner that the total heat input to all machines combined will be at a minimum. The use of incremental rates to obtain this objective is illustrated for two condensing units with automatic extraction at one point, the performance curves of which are shown in Fig. 82.

Referring to Fig. 82, it is to be noted that the heat supplied by the boilers to the units is plotted as a function of the generator load with the total heat in the steam extracted as the parameter. The use of the curves in this form will require an additional calculation for the conversion of the boiler output and turbine extraction from heat units to flow in pounds per hour. The steam generated per hour is obtained by dividing the boiler output, expressed in Btu per hour, by the difference between the enthalpies of the steam leaving and the feedwater entering the boiler. The steam extracted per hour is obtained by dividing the total heat in the extraction steam by its enthalpy.

The problem of load division for these units involves the determination of how much electric generation and extraction should be assigned

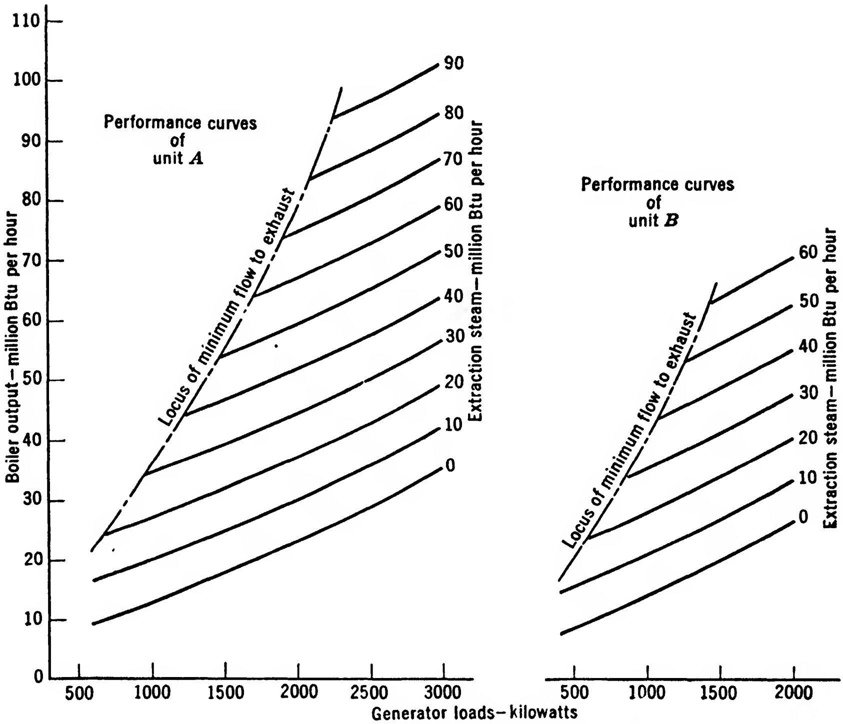


FIG. 82. Performance curves for two extraction turbine-generator units.

to each. Since the amount of extraction is independent of the amount of electric generation, the minimum boiler output for both units combined will be obtained when

$$\frac{\partial O_a}{\partial L_a} = \frac{\partial O_b}{\partial L_b} \tag{90}$$

and

$$\frac{\partial O_a}{\partial E_a} = \frac{\partial O_b}{\partial E_b} \tag{91}$$

where L_a and L_b = electric loads on units A and B, respectively, in kilowatts.

E_a and E_b = extraction from units A and B, respectively, in Btu per hour.

O_a and O_b = total boiler output chargeable to units A and B, respectively, in Btu per hour.

Let O = total boiler output chargeable to both units, in Btu per hour.

L = total electric load on both units, in kilowatts.

E = total extraction from both units, in Btu per hour.

Then

$$O = O_a + O_b$$

$$L = L_a + L_b$$

$$E = E_a + E_b$$

O will be a minimum when

$$dO = \frac{\partial O}{\partial L_a} dL_a + \frac{\partial O}{\partial E_a} dE_a = 0$$

This means that $\partial O/\partial L_a = 0$ and $\partial O/\partial E_a = 0$, hence

$$\frac{\partial O_a}{\partial L_a} + \frac{\partial O_b}{\partial L_a} = \frac{\partial O_a}{\partial L_a} + \frac{\partial O_b}{\partial L_b} \times \frac{\partial L_b}{\partial L_a} = 0$$

But

$$L_b = L - L_a$$

so

$$\frac{\partial L_b}{\partial L_a} = -1$$

and

$$\frac{\partial O_a}{\partial L_a} - \frac{\partial O_b}{\partial L_b} = 0$$

or

$$\frac{\partial O_a}{\partial L_a} = \frac{\partial O_b}{\partial L_b} \qquad [90]$$

Equation 91 can be demonstrated in a similar fashion.

TABLE XLII

INCREMENTAL LOADING OF TWO INDUSTRIAL TURBINE-GENERATORS

Extraction, Million Btu per Hour			Electric Generation, Kw		
Total	Unit A	Unit B	Total	Unit A	Unit B
10	5.5	4.5	1,000	600	400
20	10.5	9.5	1,500	1,100	400
30	16.0	14.0	2,000	1,600	400
40	21.0	19.0	2,500	1,800	700
50	27.0	23.0	3,000	2,040	960
60	32.0	28.0	3,500	2,300	1,200
70	38.0	32.0	4,000	2,550	1,450
80	43.0	37.0	4,500	2,820	1,680
90	49.0	41.0	4,850	3,000	1,850
100	55.0	45.0	5,000	3,000	2,000
110	61.0	49.0			
120	67.0	53.0			
130	73.0	57.0			
139	79.0	60.0			
145	85.0	60.0			
150	90.0	60.0			

The incremental rates for the components of boiler output chargeable to electric generation and extraction are shown for the two units in question in Fig. 83, from which the division of the total electric loads and extractions can be obtained in the manner previously described. This is shown in Table XLII.

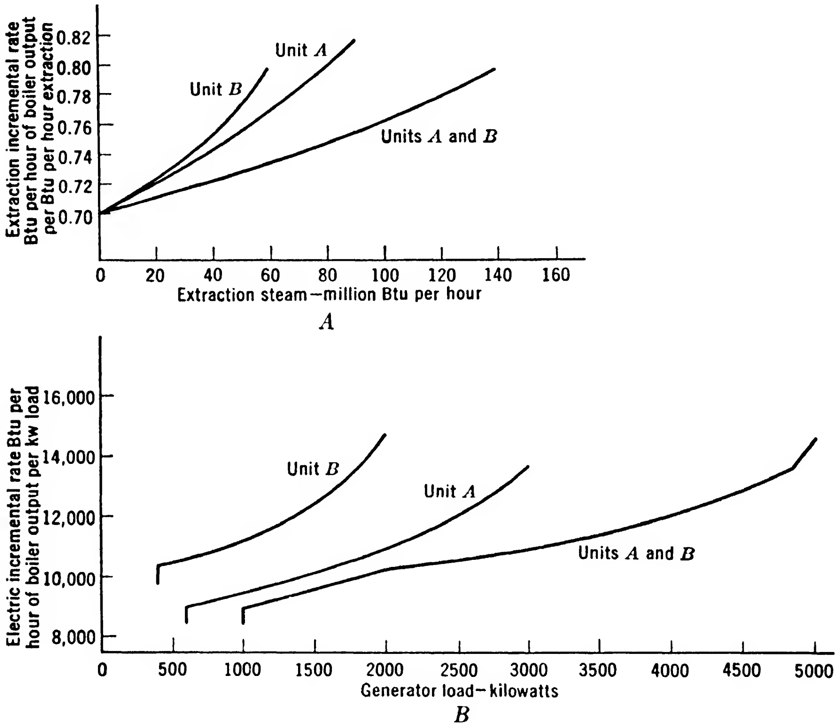


FIG. 83. *A*. Extraction incremental rate curves for the units of Fig. 82. *B*. Electric incremental rate curves for the units of Fig. 82.

It is of interest to note that the curves of Fig. 82 are parallel or very nearly so. This means that the slope of each curve for any value of load is the same and therefore independent of the quantity of extraction steam, thereby accounting for the fact that a single incremental rate curve is shown for each unit in Fig. 83*B*. If the boiler output is plotted as a function of the heat in the extraction steam, with the electric load as the parameter, a similar relation is obtained, so that a single incremental rate curve is obtained which is independent of the electric load, as shown in Fig. 83*A*. For the type of units under consideration it will generally be possible to establish single incremental rate curves without appreciable impairment of accuracy, thus simplifying the incremental loading of the units.

When the input-output curves are parallel, the boiler output may be considered to be divided into two components, one due to the electric load and the other to the extraction steam. For example, referring to the curves for unit A in Fig. 82, at a load of 1,800 kw and extraction of 20 million Btu per hour, the boiler output is 35 million Btu per hour. For the same electric load without extraction, the boiler output would be 21 million Btu per hour. Hence, the effect of the extraction is to increase the boiler output by 14 million Btu per hour, so that

Component of boiler output for electric generation	$= 21 \times 10^6$ Btu per hour
Component of boiler output for extraction	$= 14 \times 10^6$
Total boiler output	35×10^6 Btu per hour

For this situation equations 90 and 91 can be replaced respectively by

$$\frac{dO_{La}}{dL_a} = \frac{dO_{Lb}}{dL_b}. \qquad [92]$$

and

$$\frac{dO_{Ea}}{dE_a} = \frac{dO_{Eb}}{dE_b} \qquad [93]$$

where O_{La} and O_{Lb} = boiler outputs chargeable to the electric loads L_a and L_b, respectively, in Btu per hour.

O_{Ea} and O_{Eb} = boiler outputs chargeable to the extractions E_a and E_b, respectively, in Btu per hour.

It is common practice to establish the performance of extraction turbines in the form shown in Fig. 84. The use of these curves as a basis of loading the units is not recommended, because they do not take into account any differences in the enthalpies of the steam, especially in the extraction steam. However, if their use will give results with acceptable accuracy, further simplification is obtained. This follows from the fact that the throttle-flow curves are generally represented as parallel straight-line functions. Since the slope of a straight line is constant, the incremental rates will have constant values. There is then no need to plot incremental rate curves similar to those of Fig. 83, and the units are brought up to maximum load and maximum extraction in ascending order of their respective incremental rate values.

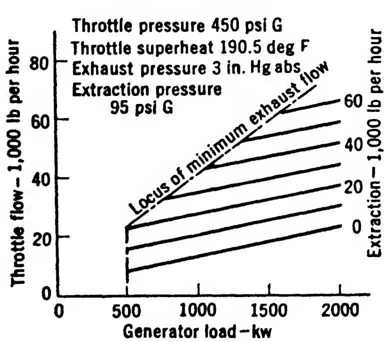


FIG. 84. Throttle flow vs. generator load for different quantities of extraction steam.

If performance curves similar to those of Fig. 84 are used, corrections should be applied for auxiliary steam consumption and requirements for heating the boiler feedwater. This adjustment is unnecessary if the performance curves are established in the form shown in Fig. 82.

There are some definite limitations to the incremental loading of extraction units which require consideration. All turbines require a certain minimum steam flow to exhaust to prevent excessive temperatures in the lower stages beyond the point of extraction. This may limit the quantity of steam that can be extracted.

In Fig. 82, the loci of minimum flow to the turbine exhausts are indicated. How this condition may limit the incremental loading of the units is illustrated by the following example.

Assume a total electric load of 3,500 kilowatts with extraction steam requirements of 117 million Btu per hour. Using the curves of Fig. 83, the incremental loading would be as follows:

	Unit *A*	Unit *B*	Total
Electric load, kw	2,300	1,200	3,500
Extraction, 10^6 Btu per hour	65	52	117

By inspection of the curves of Fig. 82 it is seen that the combination of electric load and extraction steam for unit *A* is permissible in that there would be a flow to the turbine exhaust in excess of the minimum requirement. The combination for unit *B* is not permissible. With an electric load of 1,200 kilowatts on this unit, the maximum permissible boiler output is 50 million Btu per hour, this value being obtained from the locus of minimum flow to the turbine exhaust. By interpolation the maximum permissible extraction is only 46 million Btu per hour. The incremental loading must therefore be modified to maintain the required minimum flow to the exhaust of unit *B*, which can be done in two ways.

First, divide the electric load incrementally and adjust the quantities of extraction as follows:

	Unit *A*	Unit *B*	Total
Electric load, kw	2,300	1,200	3,500
Extraction, 10^6 Btu per hour	71	46	117
Boiler output, 10^6 Btu per hour	79.23	50.00	129.23

Second, divide the extraction incrementally and adjust the electric loads as follows:

	Unit *A*	Unit *B*	Total
Extraction, 10^6 Btu per hour	65	52	117
Electric load, kw	2,180	1,320	3,500
Boiler output, 10^6 Btu per hour	73.11	56.00	129.11

The exact minimum, which will correspond to a loading combination in between these two, is difficult to determine in practice, but an average of the two loadings indicated will be sufficiently close.

Superimposed Turbine-Generators

The modernization of plants, operating at pressures ranging from 200 to 300 pounds per square inch, by superimposing a high-pressure, high-temperature turbine-generator exhausting into existing steam headers, has resulted in new problems of load division. This is especially true when part or all of the steam from the exhaust of the superimposed turbine is used for district heating. In several large metropolitan areas, where utilities supply both electric energy and steam for district heating, the original source of supply for the electric load was from one or more steam-electric plants, while steam for district heating was supplied from one or more steam-generating plants, each group of plants operating independently of the other.

Superposition in the electric plant, with provisions for utilizing steam from the exhaust of the high-pressure turbine for district heating, effects a reduction in the production costs to supply both the steam and electric loads. In addition it provides capacity for both the steam and electric loads and reduces the reserve requirements for the combined loads by reason of the diversity in time of occurrence of the respective steam and electric peak demands, thereby reducing the capital investment required.

The net effect of superposition under these circumstances is to parallel the electric and steam plants through the superimposed units, creating the problem of load division among these plants. With respect to the superimposed unit, the problem is to determine the proper allocation of the exhaust steam for electric generation by the low-pressure units which it normally supplies, and for district heating. For this purpose it is essential to understand what constitutes the incremental rate of the electric energy obtained from the exhaust steam, and the incremental rate of the exhaust steam to the street.

Figure 85 shows a heat balance for a typical superposed installation with provision for supplying steam to the street. The input-output curves for this installation with the corresponding steam and electric incremental rate curves are shown in Fig. 86. The incremental heat rate chargeable to the street steam depends on how this steam is supplied. With maximum steam flow through the topping unit, steam can be delivered to the street only by being diverted from the throttles of the topped turbines. This effects a reduction in the electric generation by these turbines and a corresponding increase in generation by other electric plants in the system or by the units in the same plant supplied with steam

from low-pressure boilers. The input to the high-pressure boilers supplying steam to the topping unit is not changed, and hence the incremental rate chargeable to the street steam supplied from the exhaust of the topping turbine is measured by the rate of increase in input to the electric plants whose electric generation has thereby been increased.

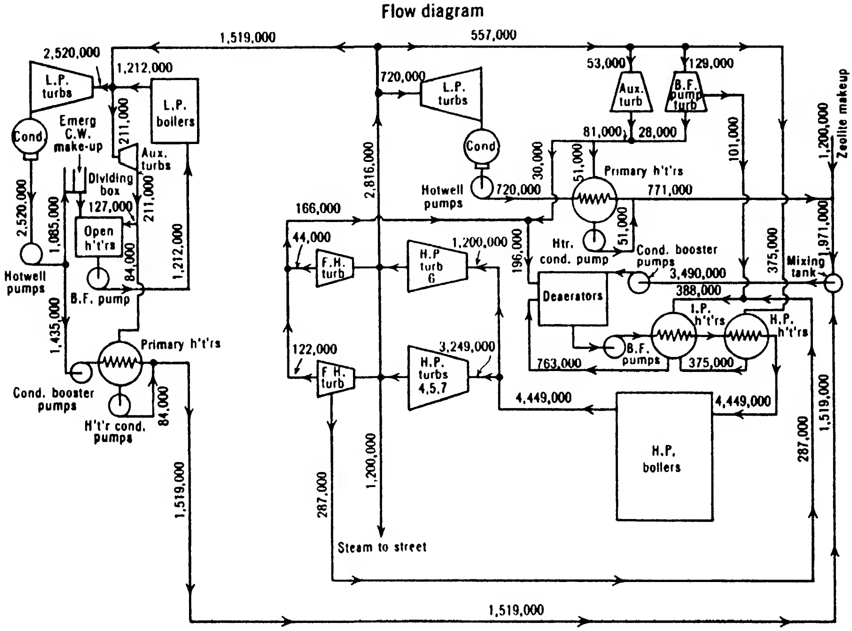


Fig. 85. Flow diagram for a high-pressure topping installation designed to supply steam for district heating.

With less than maximum steam flow through the topping turbine, street steam can be supplied from the exhaust of that unit in the following ways.

1. Steam can be diverted from the throttles of the topped turbines with a transfer of generation from these units to other stations, as described above.

2. Electric generation can be transferred from the topped units to the topping unit in amount sufficient to increase the steam flow through this unit to provide the desired quantity of street steam. The combined electric generation remains unchanged, but the input to the high-pressure boilers is increased, and this increase is chargeable to the street steam.

3. Electric generation can be transferred from other stations to the topping unit. In this instance the generation of the topped units is unchanged. The steam flow through the topping unit is increased to supply

the street steam, thereby increasing the high-pressure boiler input. The input chargeable to the street steam is the differential between the increase in input to the high-pressure boilers and the decrease in input to those electric plants from which the generation was transferred to the topping unit.

The incremental rate curves for street steam and the electric generation can be derived from the input-output data shown in Fig. 86*A*.

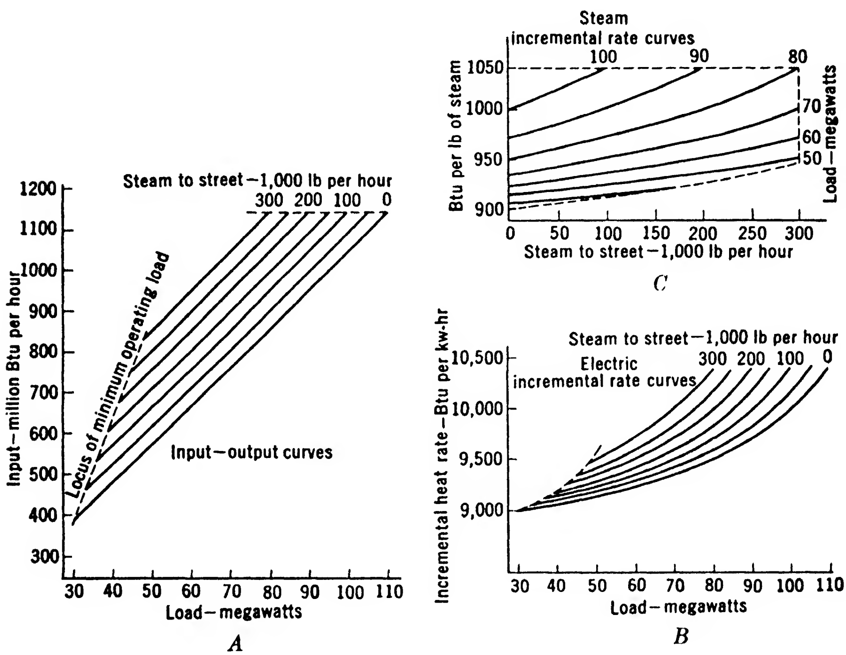


FIG. 86. Performance curves for a high-pressure topping unit supplying exhaust steam for district heating and electric generation.

The electric incremental rate curves shown in Fig. 86*B* are plotted with the street steam as the parameter. These indicate the incremental rate for any change in electric generation for any quantity of steam supplied to the street. Likewise, the incremental rate curves for the street steam, shown in Fig. 86*C*, are plotted with the electric generation as the parameter. Certain physical limits are to be noted on the curves with respect to the amount of electric generation and the quantity of steam that can be supplied to the street simultaneously. For example, referring to Fig. 86*A*, with maximum steam flow through the topping unit, in order to supply 300,000 pounds of steam per hour to the street, the electric generation would be limited to a maximum value of 80 megawatts. To increase

the electric generation would require a decrease in the quantity of steam to the street. Thus if the generation is increased to 90 megawatts, the street steam would have to be decreased to 200,000 pounds per hour. Furthermore, for any quantity of steam to the street, there is a minimum value of electric generation. For example, when 300,000 pounds of steam per hour is supplied to the street, the electric generation cannot be less than 48 megawatts.

The method for dividing a combined steam and electric load incrementally among three stations is illustrated in Fig. 87. Station A is an electric plant with a superposed installation which permits the supply of street steam from the exhaust of the topping turbine. The performance characteristics of this installation are indicated by the curves of Fig. 86. Station B is an electric plant without facilities for supplying street steam. Its incremental heat rate curve is shown in Fig. 87A. Station C is a steam plant capable of supplying street steam only. Its incremental rate curve is shown in Fig. 87B.

Since the steam and electric loads are independent variables, the curves of Fig. 87A may be used to determine the electric loading. From these it is seen that station A should be base loaded; i.e., until station A is operated at full load, station B will be operated at its minimum load. Thereafter, station B will supply any increment of system load. With the electric load division established, it is then possible to establish the steam incremental rates for station A with the system electric load as the parameter as shown by the curve net at the left of Fig. 87B. It must be remembered, however, that when station A is operating at full load electrically steam can be supplied to the street only by reducing the electric output of this station and increasing the output of station B by a corresponding amount. To illustrate, assume a total system load of 260 megawatts. From Fig. 87A the load division would be 110 megawatts on station A and 150 megawatts on station B. Suppose that it is desired to supply 50,000 pounds of steam per hour to the street from station A. It would then be necessary to reduce the electric load on station A to 105 megawatts and to increase the load on station B to 155 megawatts. The incremental load of 5 megawatts can be supplied from station B at an average incremental rate value of 11,800 Btu per kilowatt-hour, obtained from the curve for station B in Fig. 87A. The incremental input is then 5,000 kilowatts $\times$ 11,800 Btu per kilowatt-hour or 59 $\times$ 10^6 Btu per hour, and the steam incremental rate, average for the block of 50,000 pounds per hour, is 59 $\times$ 10^6 Btu per hour $\div$ 50,000 pounds per hour or 1,180 Btu per pound. By the same procedure for other values of street steam, sufficient points are calculated to establish the curve for a system electric load of 260 megawatts.

The steam incremental rate curves for stations A and C combined are shown by the curve net at the right of Fig. 87B. These are obtained in

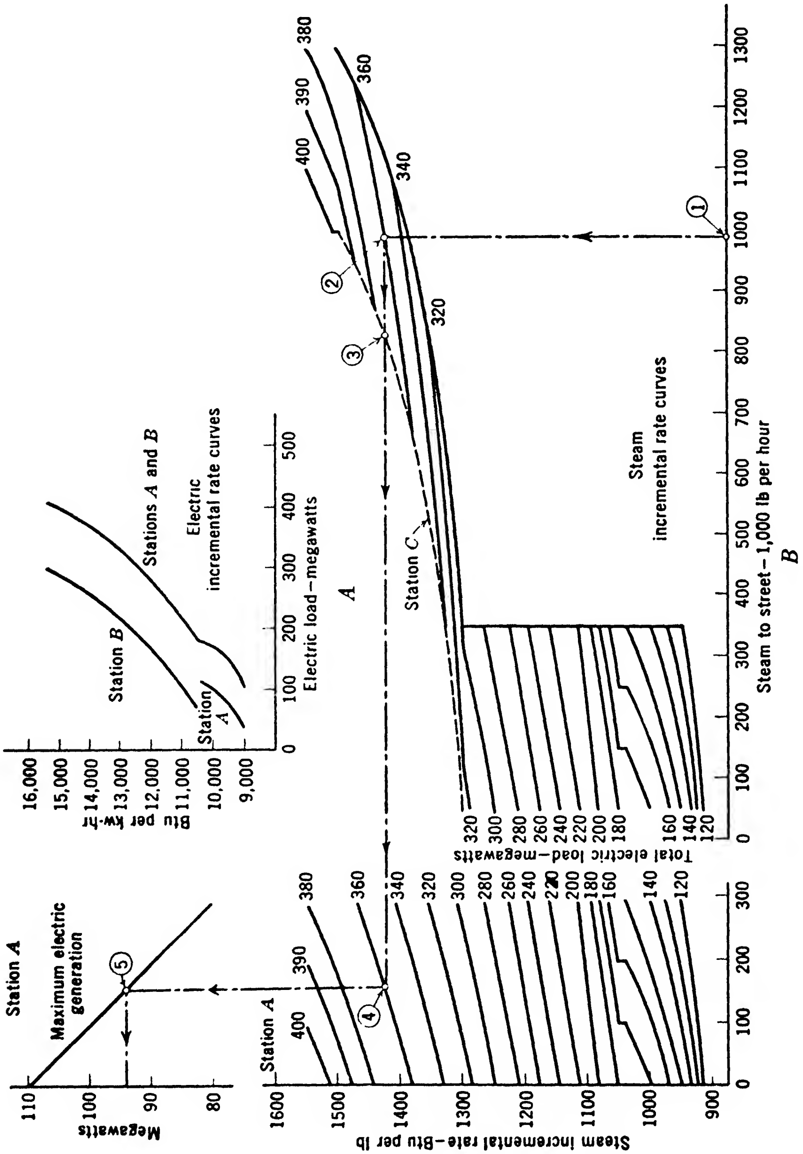


FIG. 87. Incremental rate curves for the allocation of steam and electric loads.

the usual manner by combining the curve for station C with each curve for station A.

To illustrate the use of the curves of Fig. 87, assume a system electric

load of 360 megawatts and a steam load of 990,000 pounds per hour. The load division would then be as follows:

Station	A	B	C	System
Steam load, 1,000 lb per hour	160	0	830	990
Electric load, mw	94	266	0	360

The above are obtained by entering Fig. 87B at point 1, which corresponds to the system steam demand of 990,000 pounds per hour. Point 2 on the curve corresponding to the system electric load indicates the steam incremental rate at which stations A and C should be operating. Point 3 indicates the load on station C, and point 4 the steam load on station A. The electric load on station A is indicated by point 5. The electric load on station B is then obtained as the difference between the system electric load and that supplied from station A.

Billing Priorities

The interconnection of one system with two or more other systems for the interchange of energy often results in complications with respect to determination of the proper incremental costs involved. This is best illustrated by considering a hypothetical example in which one system is interconnected with three others. Graphically, the factors to be considered are illustrated in Fig. 88. Referring to Fig. 88A, suppose that system A is operating at a load L_A. Then a block of load L_B could be supplied to system B at a cost represented by point 1 on the curve. A block of load L_C could be supplied to system C at a cost represented by point 2, and similarly the cost of supplying load L_D to system D would be represented by point 3. This would prevail if one block of load only is transferred from system A.

If system A should deliver the loads L_B, L_C, and L_D simultaneously to the respective systems, the costs for each block of load would be as represented by points 1, 2, and 3, respectively, in Fig. 88B, and the average cost to system A for the combined loads transferred would be at point 4. The cost assigned to each block of load will vary, depending upon the relative position of each block with respect to the others. For example, by reversing the sequence or positions of the blocks of load, the increment cost would be as shown in Fig. 88C, without change, however, in the average cost for the combined load. Comparing the costs indicated in Fig. 88B with the corresponding costs in Fig. 88C it is seen that, by reversing the sequence, the costs for loads L_B and L_C have been increased, and that for load L_D has been decreased. From the standpoint of an equitable billing, the incremental costs assigned to each particular block of load should be based on the same sequence as the consummation of the

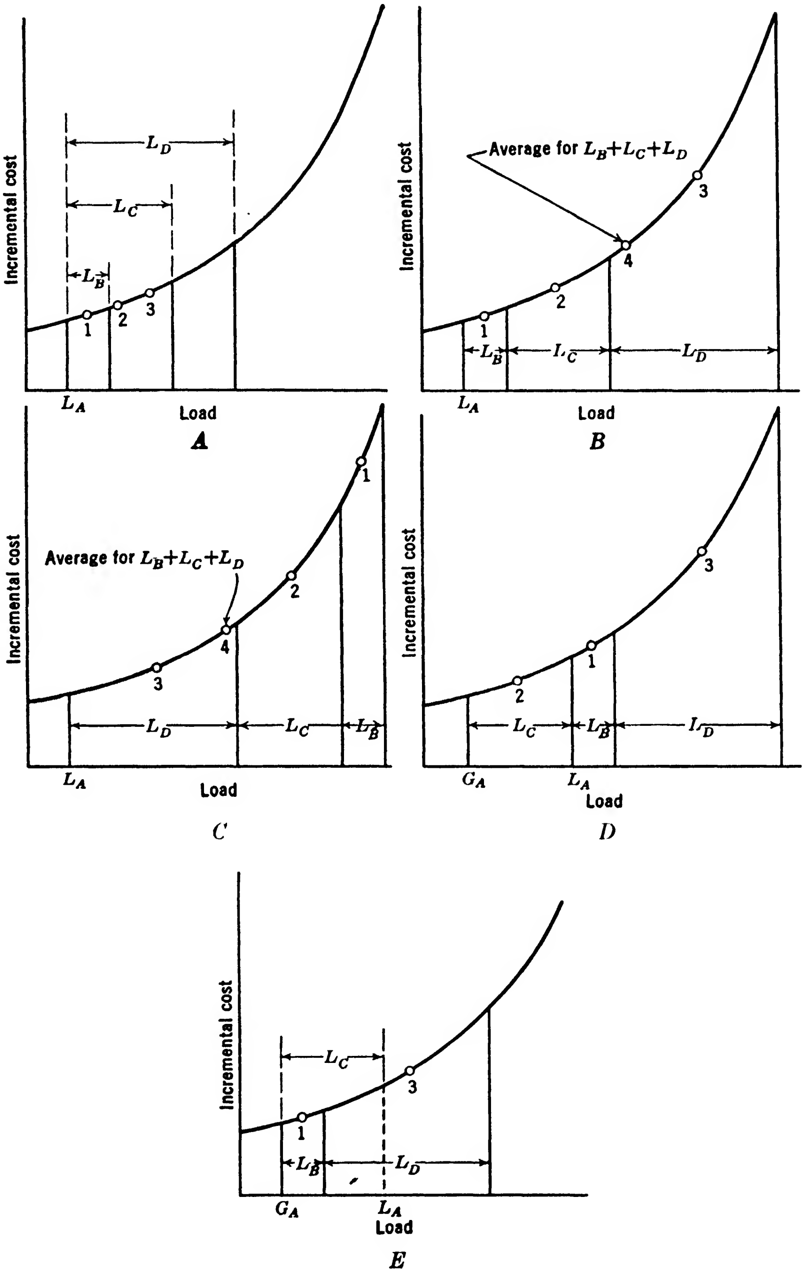


FIG. 88. Relation between billing priority and incremental production cost.

contracts. Thus Fig. 88B would represent the proper basis for determining the costs if system A first entered into a contract with system B, followed in order by a contract with system C and system D.

Suppose that system A receives energy from system C and delivers energy to systems B and D. To supply the load in its own territory system A would be required to generate $G_A = L_A - L_C$, and the decrement value of the load L_C to system A would be defined by point 2 on the curve of Fig. 88D. With respect to the incremental costs assignable to the loads L_B and L_D the values would depend on whether the blocks of load are positioned with L_A or G_A as the datum point. The two assignments are illustrated in Figs. 88D and 88E, respectively. The increment costs using G_A as the datum would be lower for each block of load than those using L_A as the datum. In other words, by using Fig. 88E as the basis, system A would be passing on to systems B and D part of the benefits derived from receiving load from system C. On the other hand, the method illustrated by Fig. 88D permits system A to retain all the benefits derived from its interchange of energy with system C, and at the same time benefit to the full extent from the delivery of energy to the other two systems by reason of the higher revenues that would result therefrom.

Effect of Spinning Reserve

Spinning reserve may be defined as the turbine-generator capacity connected to the bus in excess of the load generated. To this may be added any capacity which is available through interconnection with other systems. The spinning reserve is frequently kept equal to the capacity of the largest unit on the bus, so that the loss of any single unit will not require a reduction of the load supplied.

To supply load to an interconnection it may be necessary to add units to the bus to maintain the required spinning reserve. The cost of operating the additional units is a proper charge against the energy delivered to the interconnection. The fuel component of this cost is often measured by the increase in station input at the zero-load value resulting from the operation of the additional units. This procedure neglects the effect of the load cycle, making the cost determined on this basis incorrect.

To illustrate the principles involved, consider the input-output curves of Fig. 89 for a station consisting of four units, two rated at 160 megawatts each and two rated at 50 megawatts each. It is seen from this figure that the difference in input for successive combinations of units is not constant, being a maximum at zero-load value and gradually decreasing to negative values. When the input-output curves do not intersect, a negative difference in input will not be obtained, but the difference will generally decrease as the load is increased from zero. From the input-

output curves of Fig. 89, the difference in input can be plotted as a function of the load generated as shown in Fig. 90 for the units under consideration.

The application of the curves of Fig. 90 will first be considered for the condition when firm capacity must be provided for an interconnection.

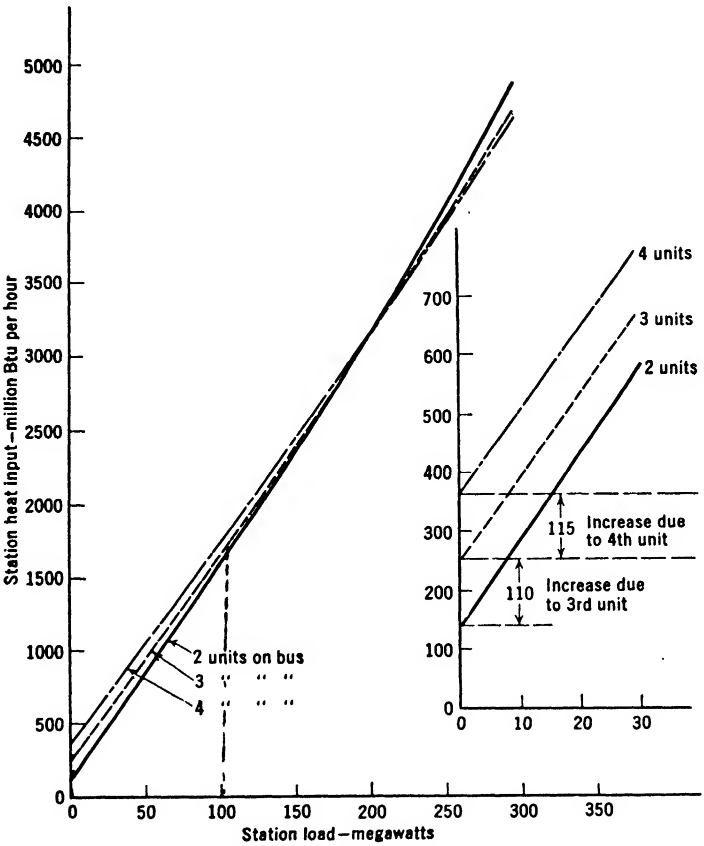


FIG. 89. Station input-output curves for successive combinations of turbine-generator units.

Figure 91 shows the daily cycle of the local demand on the plant which is to supply the firm capacity to the interconnection and the spinning capacity which would be in operation without the capacity commitment to the interconnection, which are represented by the solid lines. Assume that the plant must provide 50 megawatts of firm capacity and that this capacity must be available to the interconnection even upon the loss of the largest unit in the plant. Under these

conditions the capacity that would be available for supplying the local demand is represented by the dotted lines, so that, upon the loss of the largest unit, the cross-hatched area represents a deficiency in capacity and indicates that an additional unit is required on the station bus. The increases in station input due to the operation of the additional unit, based on the curves of Fig. 90, is shown at the top of Fig. 91. The integrated value for the day, represented by the area under the curve, gives

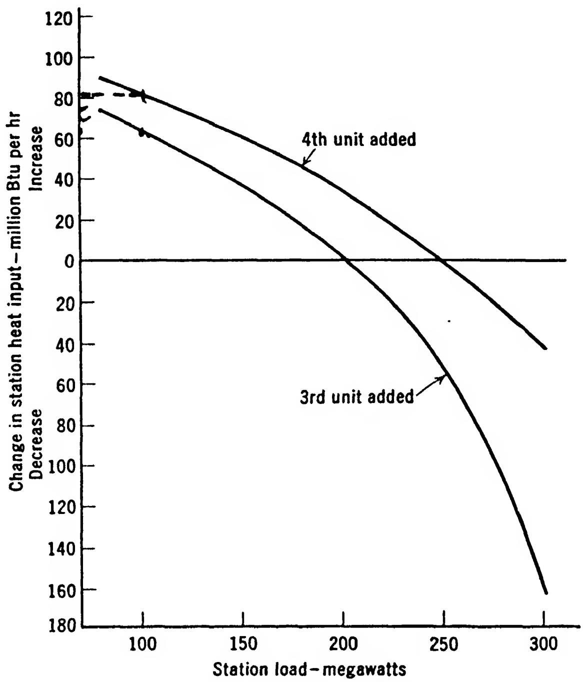


FIG. 90. Change in station heat input due to operation of an additional turbine-generator unit.

the actual input chargeable to the operation of the additional unit. Use of the zero-load input values of Fig. 89 will indicate a higher value as shown in Table XLIII, which illustrates the magnitude of the error introduced when the effect of the load cycle is neglected.

When energy is supplied to the interconnection in addition to firm capacity, the incremental input to the station supplying both may be divided into two components, one chargeable to the capacity and the other to the energy. This is illustrated in Fig. 92 for the same station under consideration. Assuming that the station is operating at a load of 110 megawatts supplied from two units, the addition of a third unit

increases the input at that load by 59 million Btu per hour. If the interconnection is supplied with 30 megawatts, the station generation will be

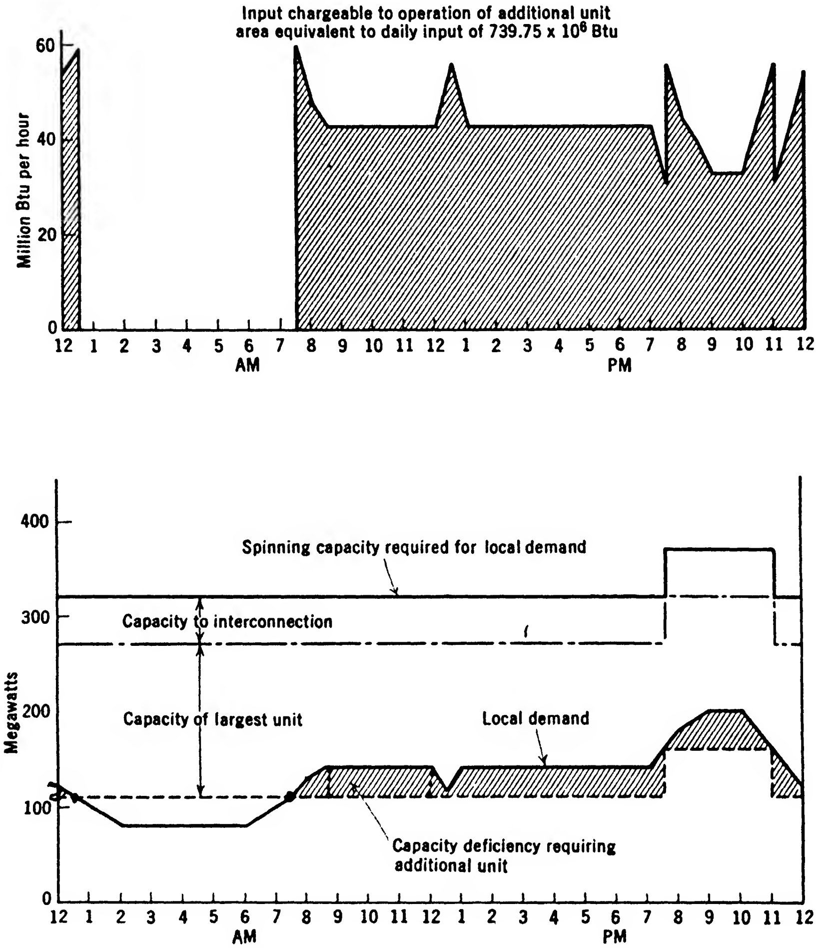


FIG. 91. Effect of spinning reserve on station heat input.

increased to 140 megawatts, and the corresponding increase in input is 430 million Btu per hour. So that

Input due to providing capacity	=	59×10^6 Btu per hour
Input due to supplying 30 mw	=	$\underline{430 \times 10^6}$
Total input		489×10^6 Btu per hour

For the above illustration, economy interchange of energy would normally be based on the incremental rates for the three-unit combina-

tion, the values of which would constitute the energy charge. In addition there would be the charge for providing firm capacity, calculated as indicated above.

TABLE XLIII

COMPARISON OF METHODS FOR CALCULATING COST OF OPERATING AN ADDITIONAL TURBINE-GENERATOR UNIT

Method A. Using Zero-Load Inputs		
Hours of service per day from Fig. 91		
3rd unit	13.5	
4th unit	3.5	
Total	17.0	
Zero-load input per hour from Fig. 89		
3rd unit	110×10^6 Btu	
4th unit	115×10^6 Btu	
Input per day chargeable to additional units		
3rd unit ($13.5 \times 110 \times 10^6$)	$1{,}485.0 \times 10^6$ Btu	
4th unit ($3.5 \times 115 \times 10^6$)	402.5	
Total	$1{,}887.5 \times 10^6$ Btu	
Calculated cost per day		
Coal @ $5 per long ton and 14,000 Btu per lb		$301
Method B. Using Curves of Fig. 90		
Input per day chargeable to additional units as indicated in Fig. 91	739.75×10^6 Btu	
Actual cost per day		
Coal @ $5 per long ton and 14,000 Btu per lb		$118
Difference per day		$183

Loading Slide Rule

On a many-station system considerable computation is required to determine the station loadings whenever changes in system conditions occur. The more important ones are outage of equipment, effect of seasonal variation in condenser circulating water temperatures, and variation in relative fuel prices. Each change requires the preparation of numerous loading schedules involving from 4 to 8 man-hours for the preparation of each schedule. In an effort to simplify the procedure several mechanical devices have been designed for load division purposes, references to descriptions of which are included in the bibliography.

A station-loading slide rule,[1] developed for the Consolidated Edison System Companies, has clearly demonstrated its utility over a period of

[1] "Station Loading Slide Rule" by H. H. Johnson and M. S. Umbenhauer, *Power*, November, 1938. "An Effective Loading Device" by H. H. Johnson and M. S. Umbenhauer, *E. E. I. Bulletin*, August, 1939.

years. Its use has permitted improvement in production economy through a closer following of changing conditions; it has provided a faster and simpler means of loading the individual stations by eliminating the need of preparing loading schedules in advance of requirement; it has also been very useful in connection with engineering studies to determine the

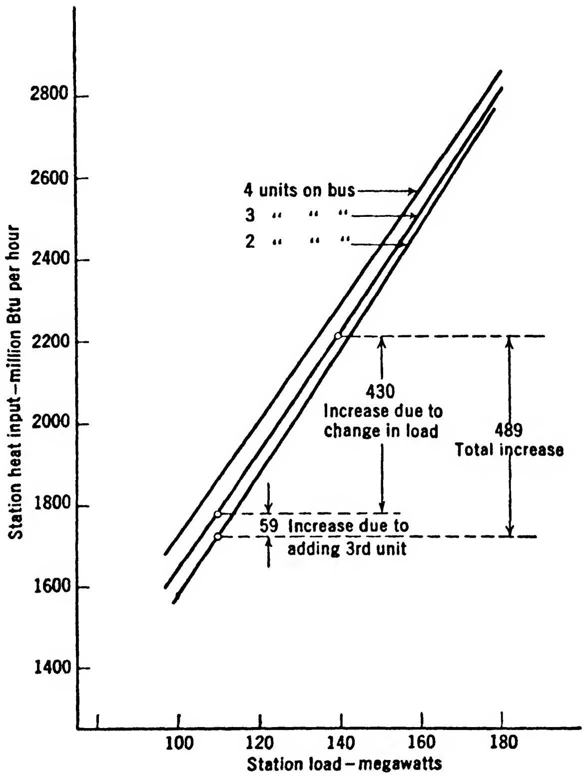


FIG. 92. Incremental heat input allocated to incremental generation and spinning reserve.

relative economies that might be expected from new installations when the final answer must be predicated upon the generation expected of the equipment.

The station-loading slide rule employs strip charts, one for each station, upon which is indicated the relation between the load generated and the incremental heat rate. The strips are prepared in the manner illustrated in Fig. 93. The incremental heat rates are calculated in steps representing average values between successive turbine-generator valve loads. For each incremental rate value a line is drawn on the strip chart. The positions of the respective lines on the strip chart correspond to the

incremental rate values as determined from the logarithmic scale at the left. The numbers adjacent to the lines on the strip chart represent the valve loads in megawatts. Thus for loads between 241 and 276 megawatts the incremental rate of 14,250 Btu per hour per kilowatt is represented by the line adjacent to the number 276, the position of which

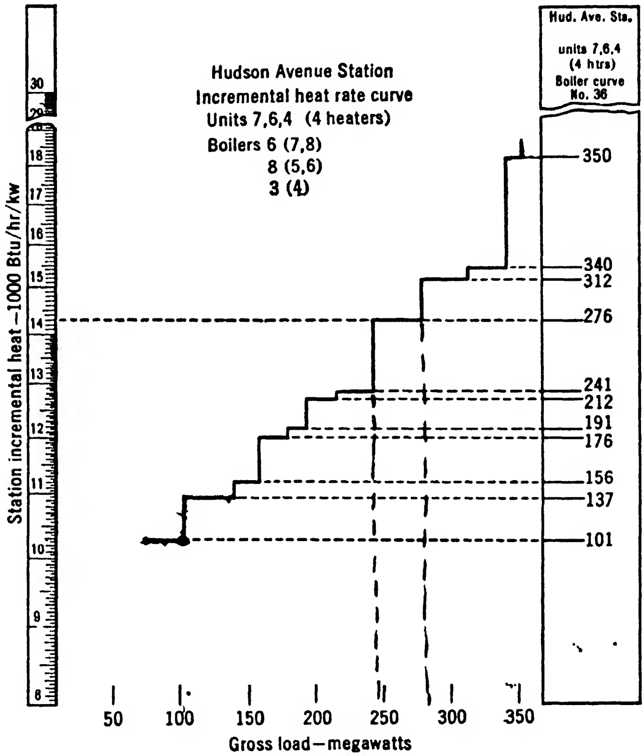


FIG. 93. Preparation of strip chart for loading slide rule.

with reference to the base is the same as that of the incremental rate value on the logarithmic scale. A separate strip is prepared for each combination of boilers and turbines expected to be operated in the station.

Because a logarithmic scale is used, it is possible to adjust for fuel price differentials at the various stations. This is illustrated in Fig. 94, which shows strips in position for loading three stations having different fuel prices. The adjustment is made by moving the strip upward until the bottom of the strip rests on the coal adjustment value which represents the ratio of the fuel prices. This is equivalent to multiplying the incremental heat rates by the fuel correction factor and therefore conforms to the procedure previously discussed.

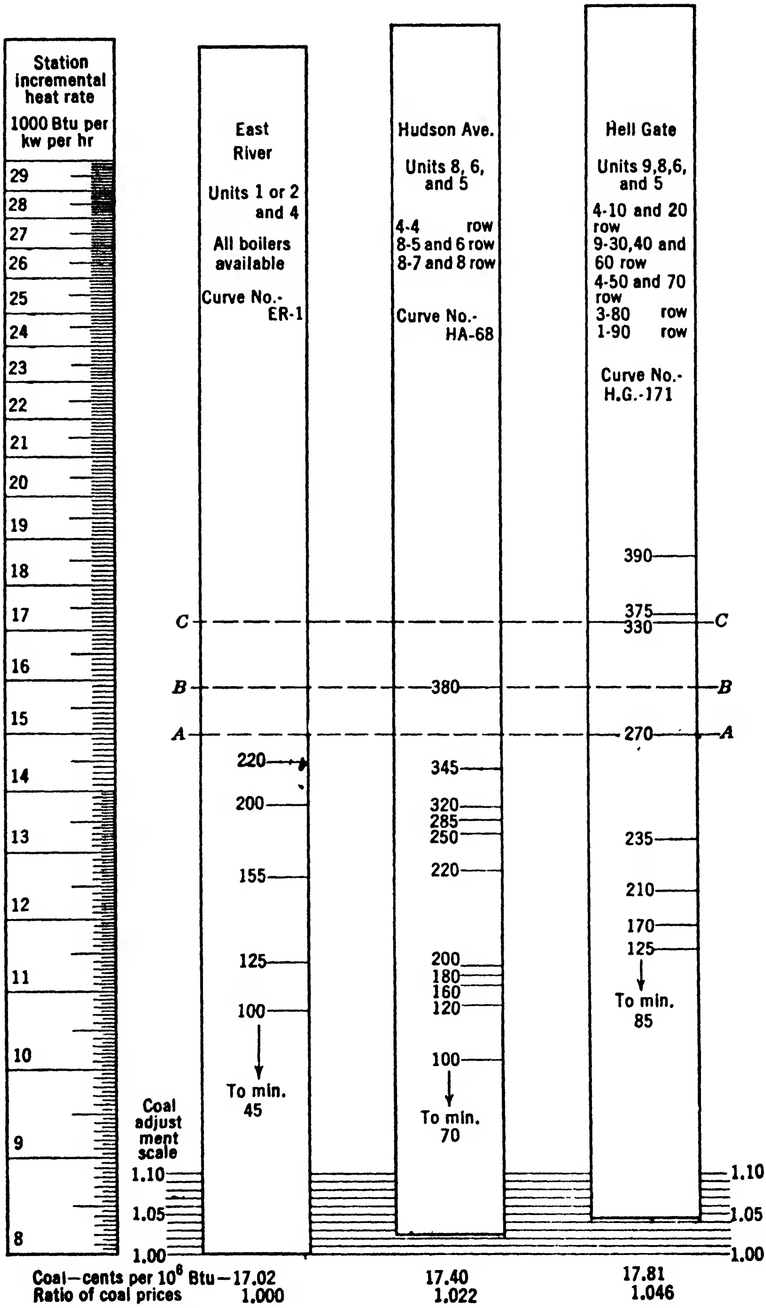


FIG. 94. Strip charts for three generating stations positioned for fuel price differentials.

By using incremental rate values in steps, only one station at a time would be affected by a change in system load. For example, if the combined load on the three stations of Fig. 94 is 835 megawatts, the load

FIG. 95. A general view of the loading slide rule.

division would be indicated by the position of line A–A. If the total load increases, the next position of the line is at B–B, which indicates that Hudson Avenue should supply the increase in load until it supplies a

oad of 380 megawatts, corresponding to a system load of 870 megawatts. Additional increases in system load to a total value of 930 megawatts would then be supplied by Hell Gate, as indicated by the position of the ine C–C. The station-loading slide rule in actual use is shown in Fig. 95.

FIG. 96. The loading slide rule with strip chart indicating incremental production cost.

Figure 96 is a view of the slide rule set up to show the strip which indicates the incremental production costs. The strips for the individual stations are shown in positions relative to a base fuel price of 15.85 cents per million Btu. The fuel correction factors are indicated by the positions of the bottoms of the strips and are based on the following fuel prices:

	Fuel Price Cents per Million Btu	Correction Factor
Base	15.85	1.00
East River	17.28	1.09
Hell Gate	17.59	1.11
Hudson Ave.	17.75	1.12

Fig. 97. The loading slide rule with strip chart indicating incremental production costs of steam supplied from the exhausts of topping turbines.

The incremental cost strip can be moved to indicate incremental fuel or production costs. When the bottom of this strip is set in line with the base incremental heat rate value of 8,000 Btu per hour per kilowatt, the

cale will indicate incremental fuel costs. The per cent adjustment scale hown at the bottom of this strip allows for the addition of a fixed percenage of the incremental fuel costs to give the incremental production cost.

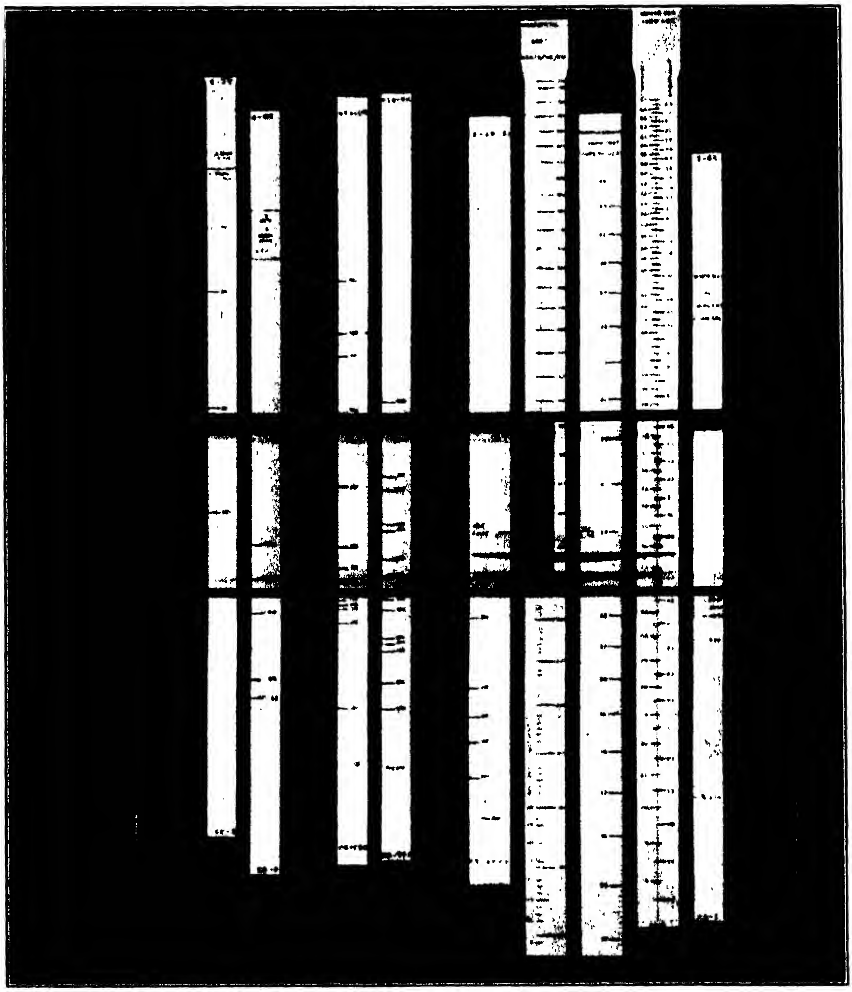


FIG. 98. The loading slide rule set up for allocating steam and electric loads.

In Fig. 96, the strip is shown for an addition of 20 per cent to the incremental fuel costs. The adjustable indicator for the fuel price scale can be set for any base fuel price. Here it is shown at the base fuel price of 15.85 cents per million Btu. As set up in Fig. 96, the loading slide rule indicates a system incremental heat rate of 15,500 Btu per kilowatt-hour and an incremental production cost of 0.295 cent per kilowatt-hour.

This value is derived by calculation as follows:

$$15{,}500 \text{ Btu/kw-hr} \times 15.85¢/10^6 \text{ Btu} \times 1.20 = 0.295¢/\text{kw-hr}$$

Figure 97 shows the slide rule with two additional strips, indicating the incremental heat rate for electric generation and the incremental cost of supplying exhaust steam to the street from the topping turbines at the Waterside Station.

Figure 98 shows a complete set-up for loading the electric and steam plants of the system. The last three strips indicate, respectively, the cost of supplying steam to the street from the exhausts of the topping turbines at Waterside; the price of steam supplied from boilers in the electric plants and from boilers in the steam plants; and the incremental heat rates for steam delivered from the Kips Bay Station of the New York Steam Corporation.

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